

Implications of residual CP symmetry for leptogenesis in two right-handed neutrino model

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Abstract

We analyze the interplay between leptogenesis and residual symmetry in the framework of two right-handed neutrino model. Working in the flavor basis, we show that all the leptogenesis CP asymmetries are vanishing for the case of two residual CP transformations or a cyclic residual flavor symmetry in the neutrino sector. If a single remnant CP transformation is preserved in the neutrino sector, the lepton mixing matrix is determined up to a real orthogonal matrix multiplied from the right side. The R -matrix is found to depend on only one real parameter, it can take three viable forms, and each entry is either real or pure imaginary. The baryon asymmetry is generated entirely by the CP violating phases in the mixing matrix in this scenario. We perform a comprehensive study for the $\Delta(6n^2)$ flavor group and CP symmetry which are broken to a single remnant CP transformation in the neutrino sector and an abelian subgroup in the charged lepton sector. The results for lepton flavor mixing and leptogenesis are presented.

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1 Introduction

A large amount of experiments with solar, atmospheric, reactor and accelerator neutrinos have provided compelling evidences for oscillations of neutrinos caused by nonzero neutrino masses and neutrino mixing [1–3]. Both three flavor neutrino and antineutrino oscillations can be described by three lepton mixing angles θ_{12} , θ_{13} and θ_{23} , one leptonic Dirac CP violating phase δ , and two independent mass-squared splittings $\delta m^2 \equiv m_2^2 - m_1^2 > 0$ and $\Delta m^2 \equiv m_3^2 - (m_1^2 + m_2^2)/2$, where $m_{1,2,3}$ are the three neutrino masses, $\Delta m^2 > 0$ and $\Delta m^2 < 0$ correspond to normal ordering (NO) and inverted ordering (IO) mass spectrum respectively. All these mixing parameters except δ have been measured with good accuracy [4–8], the experimentally allowed regions at 3σ confidence level (taken from Ref. [4]) are:

$$\begin{aligned} 0.259 &\leq \sin^2 \theta_{12} \leq 0.359, \\ 1.76(1.78) \times 10^{-2} &\leq \sin^3 \theta_{13} \leq 2.95(2.98) \times 10^{-2}, \\ 0.374(0.380) &\leq \sin^2 \theta_{23} \leq 0.626(0.641), \\ 6.99 \times 10^{-5} \text{eV}^2 &\leq \delta m^2 \leq 8.18 \times 10^{-5} \text{eV}^2, \\ 2.23(-2.56) \times 10^{-3} \text{eV}^2 &\leq \Delta m^2 \leq 2.61(-2.19) \times 10^{-3} \text{eV}^2 \end{aligned} \quad (1.1)$$

for NO (IO) neutrino mass spectrum. At present, both T2K [9, 10] and NO ν A [11] report a weak evidence for a nearly maximal CP violating phase $\delta \sim -\pi/2$, and hits of $\delta \sim -\pi/2$ also show up in the global fit of neutrino oscillation data [4–8]. Moreover, several experiments are being planned to look for CP violation in neutrino oscillation, including long-baseline facilities, superbeams, and neutrino factories. The above structure of lepton mixing, so different from the the small mixing in the quark sector, provides a great theoretical challenge. The idea of flavor symmetry has been extensively exploited to provide a realistic description of the lepton masses and mixing angles. The finite discrete non-abelian flavor symmetries have been found to be particularly interesting as they can naturally lead to certain mixing patterns [12], please see Refs. [13–15] for review.

Although the available data are not yet able to determine the individual neutrino mass m_i , the neutrino masses are known to be of order eV from tritium endpoint, neutrinoless double beta decay and cosmological data. The smallness of neutrino masses can be well explained within the see-saw mechanism [16], in which the Standard Model (SM) is extended by adding new heavy states. The light neutrino masses are generically suppressed by the large masses of the new states. In type I seesaw model [16] the extra states are right-handed (RH) neutrinos which have Majorana masses much larger than the electroweak scale, unlike the standard model fermions which acquire mass proportional to electroweak symmetry breaking. Apart from elegantly explaining the tiny neutrino masses, the seesaw mechanism provides a simple and attractive explanation for the observed baryon asymmetry of the Universe, one of the most longstanding cosmological puzzles. The CP violating decays of heavy RH neutrinos can produce a lepton asymmetry in the early universe, which is then converted into a baryon asymmetry through $B + L$ violating anomalous sphaleron processes at the electroweak scale. This is the so-called leptogenesis mechanism [17].

It is well-known that in the paradigm of the unflavored thermal leptogenesis the CP phases in the neutrino Yukawa couplings in general are not related to the the low energy leptonic CP violating parameters (i.e. Dirac and Majorana phases) in the mixing matrix. However, the low energy CP phases could play a crucial role in the flavored thermal leptogenesis [18] in which the flavors of the charged leptons produced in the heavy RH neutrino decays are relevant. In models with flavor symmetry, the total number of free parameters is greatly reduced, therefore the observed baryon asymmetry could possibly be related to other observable quantities [19]. In general, the leptogenesis CP asymmetries would vanish if a Klein subgroup of the flavor symmetry group is preserved in the neutrino sector [20].

Recent studies show that the extension of discrete flavor symmetry to include CP symmetry is a

very predictive framework [21–33]. If the given flavor and CP symmetries are broken to an abelian subgroup and $Z_2 \times CP$ in the charged lepton and neutrino sectors respectively, the resulting lepton mixing matrix would be determined in terms of a free parameter θ whose value can be fixed by the reactor angle θ_{13} . Hence all the lepton mixing angles, Dirac CP violating phase and Majorana CP phases can be predicted [33]. Moreover, other phenomena involving CP phases such as neutrinoless double decay and leptogenesis are also strongly constrained in this approach [20, 31, 34]. In fact, we find that the leptogenesis CP asymmetries are exclusively due to the Dirac and Majorana CP phases in the lepton mixing matrix, and the R -matrix depends on only a single real parameter in this scenario [20].

In this paper we shall extend upon the work of [20] in which the SM is extended to introduce three RH neutrinos. Here we shall study the interplay between residual symmetry and leptogenesis in seesaw model with two RH neutrinos. We find that all the leptogenesis CP asymmetries would be exactly vanishing if two residual CP transformations or a cyclic residual flavor symmetry are preserved by the seesaw Lagrangian. On the other hand, if only one remnant CP transformation is preserved in the neutrino sector, all mixing angles and CP phases are then fixed in terms of three real parameters $\theta_{1,2,3}$ which can take values between 0 and π , and the R -matrix would be constrained to depend on only one free parameter. The total CP asymmetry $\epsilon_1 \equiv \epsilon_e + \epsilon_\mu + \epsilon_\tau$ in leptogenesis is predicted to be zero. Hence our discussion will be entirely devoted to the flavored thermal leptogenesis scenario in which the lightest RH neutrino mass is typically in the interval of $10^9 \text{ GeV} \leq M_1 \leq 10^{12} \text{ GeV}$. Our approach is quite general and it is independent of the explicit form of the residual symmetries and how the vacuum alignment achieving the residual symmetries is dynamically realized. In order to show concrete examples, we apply this general formalism to the flavor group $\Delta(6n^2)$ combined with CP symmetry which is broken down to an abelian subgroup in the charged lepton sector and a remnant CP transformation in the neutrino sector. The expressions for lepton mixing matrix as well as mixing parameters in each possible cases are presented. We find that for small values of the flavor group index n , the experimental data on lepton mixing angles can be accommodated for certain values of the parameters $\theta_{1,2,3}$. The corresponding predictions for the cosmological matter-antimatter asymmetry are discussed.

The rest of the paper is organized as follows. In section 2 we briefly review some generic aspects of leptogenesis in two RH model and present some analytic approximations which will be used later. In section 3 we study the scenario that one residual CP transformation is preserved in the neutrino sector. The lepton mixing matrix is determined up to an arbitrary real orthogonal matrix multiplied from the right hand side. The R -matrix contains only one free parameter, and each element is either real or pure imaginary. The total CP asymmetry ϵ_1 is vanishing, consequently the unflavored leptogenesis is not feasible unless subleading corrections are taken into account. The scenario of two remnant CP transformations or a cyclic residual flavor symmetry is discussed in section 4. All leptogenesis CP asymmetries $\epsilon_{e,\mu,\tau}$ are found to vanish in both cases. Leptogenesis could become potentially viable only when higher order contributions lift the postulated residual symmetry. In section 5 we apply our general formalism to the case that the single residual CP transformation of the neutrino sector arises from the breaking of the most general CP symmetry compatible with $\Delta(6n^2)$ flavor group which is broken down to an abelian subgroup in the charged lepton sector. The predictions for lepton flavor mixing and baryon asymmetry are studied analytically and numerically. Finally, in section 6 we summarize our main results and draw the conclusions.

2 General set-up of leptogenesis in two right-handed neutrino model

Seesaw mechanism is a popular extension of the Standard Model (SM) to explain the smallness of neutrino masses. In the famous type I seesaw mechanism [16], one generally introduces addi-

tional three right-handed neutrinos which are singlets under the SM gauge group. Although the seesaw mechanism describes qualitatively well the observations in neutrino oscillation experiments, it is quite difficult to make quantitative predictions for neutrino mass and mixing without further hypothesis for underlying dynamics. The reason is that the seesaw mechanism involves a large number of undetermined parameters at high energies whereas much less parameters could be measured experimentally.

A intriguing way out of this problem is to simply reduce the number of right-handed neutrinos from three to two [35–37]. The two right-handed neutrino (2RHN) model can be regarded as a limiting case of three right-handed neutrinos where one of the RH neutrinos decouples from the seesaw mechanism either because it is very heavy or because its Yukawa couplings are very weak. Since the number of free parameters is greatly reduced, the 2RHN model is more predictive than the standard scenario involving three RH neutrinos. Namely, the lightest left-handed neutrino mass automatically vanishes, while the masses of the other two neutrinos are fixed by δm^2 and Δm^2 . Hence only two possible mass spectrums can be obtained

$$\begin{aligned} \text{NO : } m_1 &= 0, \quad m_2 = \sqrt{\delta m^2}, \quad m_3 = \sqrt{\Delta m^2 + \delta m^2/2}, \\ \text{IO : } m_1 &= \sqrt{-\delta m^2/2 - \Delta m^2}, \quad m_2 = \sqrt{\delta m^2/2 - \Delta m^2}, \quad m_3 = 0. \end{aligned} \quad (2.1)$$

Moreover there is only one Majorana CP violating phase corresponding to the phase difference between these two nonzero mass eigenvalues. The Lagrangian responsible for lepton masses in the 2RHN model takes the following form

$$\mathcal{L} = -y_\alpha \bar{L}_\alpha H l_{\alpha R} - \lambda_{i\alpha} \bar{N}_{iR} \tilde{H}^\dagger L_\alpha - \frac{1}{2} M_i \bar{N}_{iR} N_{iR}^c + h.c. , \quad (2.2)$$

where $L_\alpha \equiv (\nu_{\alpha L}, l_{\alpha L})^T$ and $l_{\alpha R}$ indicate the lepton doublet and singlet fields with flavor $\alpha = e, \mu, \tau$ respectively, N_{iR} is the RH neutrino with mass M_i ($i = 1, 2$), and $H \equiv (H^+, H^0)^T$ is the Higgs doublet with $\tilde{H} \equiv i\sigma_2 H^*$. The Yukawa couplings $\lambda_{i\alpha}$ form an arbitrary complex 2×3 matrix, here we have worked in the basis in which both the Yukawa couplings for the charged leptons and the Majorana mass matrix for the RH neutrinos are diagonal and real. After electroweak symmetry breaking, the light neutrino mass matrix is given by the famous seesaw formula

$$m_\nu = v^2 \lambda^T M^{-1} \lambda = U^* m U^\dagger, \quad (2.3)$$

where $v = 175$ GeV refers to the vacuum expectation value of the Higgs field H^0 , $M \equiv \text{diag}(M_1, M_2)$ and $m \equiv \text{diag}(m_1, m_2, m_3)$ with $m_1 = 0$ for NO and $m_3 = 0$ for IO, and U is the lepton mixing matrix. It is convenient to express the Yukawa coupling λ in terms of the neutrino mass eigenvalues, mixing angles and CP violation phases as¹

$$\lambda = M^{1/2} R m^{1/2} U^\dagger / v, \quad (2.4)$$

where R is a 2×3 complex orthogonal matrix having the following structure [38, 39]

$$\text{NO : } R = \begin{pmatrix} 0 & \cos \hat{\theta} & \xi \sin \hat{\theta} \\ 0 & -\sin \hat{\theta} & \xi \cos \hat{\theta} \end{pmatrix}, \quad (2.5a)$$

$$\text{IO : } R = \begin{pmatrix} \cos \hat{\theta} & \xi \sin \hat{\theta} & 0 \\ -\sin \hat{\theta} & \xi \cos \hat{\theta} & 0 \end{pmatrix}, \quad (2.5b)$$

where $\hat{\theta}$ is an arbitrary complex number and $\xi = \pm 1$. From Eqs. (2.5a, 2.5b) we can check that the R -matrix satisfies

$$\begin{aligned} R R^T &= \text{diag}(1, 1), \quad \text{for NO and IO}, \\ R^T R &= \text{diag}(0, 1, 1), \quad \text{for NO}, \\ R^T R &= \text{diag}(1, 1, 0), \quad \text{for IO}. \end{aligned} \quad (2.6)$$

¹For other parameterizations of the neutrino Yukawa coupling, see Ref. [40].

Leptogenesis is a natural consequence of the seesaw mechanism, and it provides an elegant explanation for the baryon asymmetry of the Universe [17]. We shall work in the typical N_1 -dominated scenario, and we assume that right-handed neutrinos are hierarchical $M_2 \gg M_1$ such that the asymmetry is dominantly produced from the decays of the lightest RH neutrino N_1 . The phenomenology of leptogenesis in 2RHN model has been comprehensively studied [36, 37, 39, 41]. The flavored CP asymmetries in the decays of N_1 into leptons of different flavors are of the form [42–45]

$$\begin{aligned}\epsilon_\alpha &\equiv \frac{\Gamma(N_1 \rightarrow l_\alpha H) - \Gamma(N_1 \rightarrow \bar{l}_\alpha \bar{H})}{\sum_\alpha \Gamma(N_1 \rightarrow l_\alpha H) + \Gamma(N_1 \rightarrow \bar{l}_\alpha \bar{H})} \\ &\simeq -\frac{3}{16\pi} \sum_{j=1}^2 \frac{M_1}{M_j} \frac{\Im[(\lambda\lambda^\dagger)_{1j} \lambda_{1\alpha} \lambda_{j\alpha}^*]}{(\lambda\lambda^\dagger)_{11}} \\ &= -\frac{3M_1}{16\pi v^2} \frac{\Im(\sum_{ij} \sqrt{m_i m_j} m_j R_{1i} R_{1j} U_{\alpha i}^* U_{\alpha j})}{\sum_j m_j |R_{1j}|^2},\end{aligned}\quad (2.7)$$

where $\Gamma(N_1 \rightarrow l_\alpha H)$ and $\Gamma(N_1 \rightarrow \bar{l}_\alpha \bar{H})$ with $\alpha = e, \mu, \tau$ denote the flavored decay rates of N_1 into lepton l_α and anti-lepton \bar{l}_α respectively. We notice that ϵ_α is invariant under the transformation $\xi \rightarrow -\xi$ and $\hat{\theta} \rightarrow -\hat{\theta}$. Consequently we shall choose $\xi = 1$ as an illustration in the following numerical analysis. Inserting the expression for the Yukawa coupling in Eqs. (2.5a, 2.5b) into Eq. (2.7), we obtain the CP asymmetry

$$\begin{aligned}\epsilon_\alpha &\simeq -\frac{3}{16\pi v^2} \frac{M_1}{m_2 |\cos \hat{\theta}|^2 + m_3 |\sin \hat{\theta}|^2} \left\{ (m_3^2 |U_{\alpha 3}|^2 - m_2^2 |U_{\alpha 2}|^2) \Im \sin^2 \hat{\theta} \right. \\ &\quad \left. + \xi \sqrt{m_2 m_3} \left[(m_2 + m_3) \Re(U_{\alpha 2}^* U_{\alpha 3}) \Im(\sin \hat{\theta} \cos \hat{\theta}) + (m_3 - m_2) \Im(U_{\alpha 2}^* U_{\alpha 3}) \Re(\sin \hat{\theta} \cos \hat{\theta}) \right] \right\},\end{aligned}\quad (2.8)$$

for NO and

$$\begin{aligned}\epsilon_\alpha &\simeq -\frac{3}{16\pi v^2} \frac{M_1}{m_1 |\cos \hat{\theta}|^2 + m_2 |\sin \hat{\theta}|^2} \left\{ (m_2^2 |U_{\alpha 2}|^2 - m_1^2 |U_{\alpha 1}|^2) \Im \sin^2 \hat{\theta} \right. \\ &\quad \left. + \xi \sqrt{m_1 m_2} \left[(m_1 + m_2) \Re(U_{\alpha 1}^* U_{\alpha 2}) \Im(\sin \hat{\theta} \cos \hat{\theta}) + (m_2 - m_1) \Im(U_{\alpha 1}^* U_{\alpha 2}) \Re(\sin \hat{\theta} \cos \hat{\theta}) \right] \right\}\end{aligned}\quad (2.9)$$

for IO neutrino mass spectrum. If the RH neutrino mass M_1 is large enough (e.g. $M_1 > 10^{12}$ GeV), the interactions mediated by all the three charged lepton Yukawa couplings are out of equilibrium. As a result, the one flavor approximation rigorously holds, and the total CP asymmetry is

$$\epsilon_1 \equiv \sum_\alpha \epsilon_\alpha = -\frac{3M_1}{16\pi v^2} \frac{\Im(\sum_i m_i^2 R_{1i}^2)}{\sum_j m_j |R_{1j}|^2},\quad (2.11)$$

which is completely independent of the lepton mixing matrix U . For the parametrization of the R -matrix in Eqs. (2.5a, 2.5b), we have

$$\text{NO : } \epsilon_1 = -\frac{3M_1}{16\pi v^2} \frac{(m_3^2 - m_2^2) \Im \sin^2 \hat{\theta}}{m_2 |\cos \hat{\theta}|^2 + m_3 |\sin \hat{\theta}|^2},\quad (2.12a)$$

$$\text{IO : } \epsilon_1 = -\frac{3M_1}{16\pi v^2} \frac{(m_2^2 - m_1^2) \Im \sin^2 \hat{\theta}}{m_1 |\cos \hat{\theta}|^2 + m_2 |\sin \hat{\theta}|^2}.\quad (2.12b)$$

We see that the total CP asymmetry ϵ_1 would vanish when the parameter $\hat{\theta}$ is real or pure imaginary up to $\pi/2$. The total baryon asymmetry is the sum of each individual lepton asymmetry. In the present paper we will be concerned with temperature window ($10^9 \leq T \sim M_1 \leq 10^{12}$) GeV. In

this range only the τ charged lepton Yukawa interaction is in equilibrium, the e and μ flavors are indistinguishable, and the final baryon asymmetry is well approximated by [46–49]

$$Y_B \simeq -\frac{12}{37g_*} \left[\epsilon_2 \eta \left(\frac{417}{589} \tilde{m}_2 \right) + \epsilon_\tau \eta \left(\frac{390}{589} \tilde{m}_\tau \right) \right], \quad (2.13)$$

where $\epsilon_2 \equiv \epsilon_e + \epsilon_\mu$, $\tilde{m}_2 \equiv \tilde{m}_e + \tilde{m}_\mu$ and

$$\eta(\tilde{m}_\alpha) \simeq \left[\left(\frac{\tilde{m}_\alpha}{8.25 \times 10^{-3} \text{ eV}} \right)^{-1} + \left(\frac{0.2 \times 10^{-3} \text{ eV}}{\tilde{m}_\alpha} \right)^{-1.16} \right]^{-1}. \quad (2.14)$$

Here g_* is the number of relativistic degrees of freedom, the efficiency factor $\eta(\tilde{m}_\alpha)$ accounts for the washing out of the produced lepton number asymmetries due to the inverse decay and lepton number violating scattering, and the washout mass \tilde{m}_α parametrizes the decay rate of N_1 into the leptons of flavor α with

$$\tilde{m}_\alpha \equiv \frac{|\lambda_{1\alpha}|^2 v^2}{M_1} = \left| \sum_i m_i^{1/2} R_{1i} U_{\alpha i}^* \right|^2, \quad \alpha = e, \mu, \tau. \quad (2.15)$$

Plugging Eqs. (2.5a) and (2.5b) into above equation we find the explicit expressions of the washout masses are

$$\tilde{m}_\alpha = \begin{cases} \left| \sqrt{m_2} U_{\alpha 2}^* \cos \hat{\theta} + \xi \sqrt{m_3} U_{\alpha 3}^* \sin \hat{\theta} \right|^2, & \text{for NO}, \\ \left| \sqrt{m_1} U_{\alpha 1}^* \cos \hat{\theta} + \xi \sqrt{m_2} U_{\alpha 2}^* \sin \hat{\theta} \right|^2, & \text{for IO}. \end{cases} \quad (2.16)$$

3 Leptogenesis with one residual CP transformation

In a series of papers [21–33], it has been shown that the residual CP symmetry of the light neutrino mass matrix can quite efficiently predict the lepton mixing angles as well as CP violation phases. If the residual CP symmetry is preserved by the seesaw Lagrangian, leptogenesis would be also strongly constrained [20, 34, 50]. We assume that the flavor and CP symmetries are broken at a scale above the leptogenesis scale. As a consequence, leptogenesis occurs in the standard framework of the SM plus two heavy RH neutrinos without involving any additional state in its dynamics. In this section, we shall study the implications of residual CP for leptogenesis in 2RHN model, and we assume that both the neutrino Yukawa coupling and the Majorana mass term in Eq. (2.2) are invariant under one generic residual CP transformation defined as

$$\nu_L \xrightarrow{\text{CP}} i X_\nu \gamma_0 C \bar{\nu}_L^T, \quad N_R \xrightarrow{\text{CP}} i \hat{X}_N \gamma_0 C \bar{N}_R^T, \quad (3.1)$$

where $\nu_L \equiv (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})^T$, $N_R \equiv (N_{1R}, N_{2R})^T$, C denotes the charge-conjugation matrix, X_ν is a 3×3 symmetric unitary matrix to avoid degenerate neutrino masses and \hat{X}_N is a 2×2 symmetric unitary matrix. For the symmetry to hold, λ and M have to fulfill

$$\hat{X}_N^\dagger \lambda X_\nu = \lambda^*, \quad \hat{X}_N^\dagger M \hat{X}_N^* = M^*. \quad (3.2)$$

As we work in the basis in which the RH neutrino mass matrix M is real and diagonal, the residual CP transformation \hat{X}_N should be diagonal with elements equal to ± 1 , i.e.,

$$\hat{X}_N = \text{diag}(\pm 1, \pm 1), \quad (3.3)$$

Notice that conclusion would be not changed even if M is non-diagonal in a concrete flavor symmetry model [20]. Thus we can find that the light neutrino mass matrix m_ν given by the seesaw formula satisfies

$$X_\nu^T m_\nu X_\nu = m_\nu^*, \quad (3.4)$$

which means (as expected) m_ν is invariant under the residual CP transformation X_ν . The light neutrino mass matrix can be diagonalized by a unitary transformation U_ν with $m_\nu = U_\nu^* \text{diag}(m_1, m_2, m_3) U_\nu^\dagger$. Then from Eq. (3.4) we can obtain

$$\left(U_\nu^\dagger X_\nu U_\nu^* \right)^T \text{diag}(m_1, m_2, m_3) \left(U_\nu^\dagger X_\nu U_\nu^* \right) = \text{diag}(m_1, m_2, m_3).$$

Note $m_1 = 0$ for NO and $m_3 = 0$ for IO in the 2RHN model. Hence U_ν is subject to the following constraint from the residual CP transformation X_ν ,

$$U_\nu^\dagger X_\nu U_\nu^* = \hat{X}_\nu, \quad (3.5)$$

with

$$\begin{aligned} \hat{X}_\nu &= \text{diag}(e^{i\alpha}, \pm 1, \pm 1) \quad \text{for NO}, \\ \hat{X}_\nu &= \text{diag}(\pm 1, \pm 1, e^{i\alpha}) \quad \text{for IO}, \end{aligned} \quad (3.6)$$

where α is a real parameter in the interval between 0 and 2π . Then it is easy to check that X_ν is a symmetric and unitary matrix for both NO and IO cases. Moreover, with the definition of R -matrix in Eq. (2.4), we can derive that the postulated residual symmetry leads to the following constraint on R as

$$\hat{X}_N R^* \hat{X}_\nu^{-1} = R, \quad (3.7)$$

Obviously $-\hat{X}_N$ and $-\hat{X}_\nu$ give rise to the same constraint on R as \hat{X}_N and \hat{X}_ν , therefore it is sufficient to only consider the cases of $\hat{X}_N = \text{diag}(1, \pm 1)$, $\hat{X}_\nu = \text{diag}(e^{i\alpha}, \pm 1, \pm 1)$ for NO and $\hat{X}_\nu = \text{diag}(\pm 1, \pm 1, e^{i\alpha})$ for IO. The explicit forms of the R -matrix for all possible values of \hat{X}_N and \hat{X}_ν are collected in table 1. We see that there are three admissible forms of the R -matrix summarized as follows

$$\begin{aligned} \text{R-1st:} \quad & \begin{cases} R = \begin{pmatrix} 0 & \cos \vartheta & \xi \sin \vartheta \\ 0 & -\sin \vartheta & \xi \cos \vartheta \end{pmatrix} & \text{for NO}, \\ R = \begin{pmatrix} \cos \vartheta & \xi \sin \vartheta & 0 \\ -\sin \vartheta & \xi \cos \vartheta & 0 \end{pmatrix} & \text{for IO}, \end{cases} \\ \text{R-2nd:} \quad & \begin{cases} R = \pm \begin{pmatrix} 0 & \cosh \vartheta & i\xi \sinh \vartheta \\ 0 & -i \sinh \vartheta & \xi \cosh \vartheta \end{pmatrix} & \text{for NO}, \\ R = \pm \begin{pmatrix} \cosh \vartheta & i\xi \sinh \vartheta & 0 \\ -i \sinh \vartheta & \xi \cosh \vartheta & 0 \end{pmatrix} & \text{for IO}, \end{cases} \\ \text{R-3rd:} \quad & \begin{cases} R = \pm \begin{pmatrix} 0 & i \sinh \vartheta & -\xi \cosh \vartheta \\ 0 & \cosh \vartheta & i\xi \sinh \vartheta \end{pmatrix} & \text{for NO}, \\ R = \pm \begin{pmatrix} i \sinh \vartheta & -\xi \cosh \vartheta & 0 \\ \cosh \vartheta & i\xi \sinh \vartheta & 0 \end{pmatrix} & \text{for IO}. \end{cases} \end{aligned} \quad (3.8)$$

We would like to point out that the R -matrix is constrained to depend on a single real parameter ϑ in this setup. Moreover, from Eq. (2.11) we can see that the total lepton asymmetry ϵ_1 is vanishing, i.e.

$$\epsilon_1 = \epsilon_e + \epsilon_\mu + \epsilon_\tau = 0. \quad (3.9)$$

As a result, the net baryon asymmetry can not be generated in the one flavor approximation which is realized when the mass of the lightest right-handed neutrino M_1 is larger than about 10^{12} GeV, unless the residual CP symmetry is further broken by subleading order corrections. This result is quite general, it is independent of the explicit form of the residual CP transformation and how the residual symmetry is dynamically realized.

Next we proceed to determine the lepton mixing matrix from the postulated remnant CP transformation. Since X_ν must be a symmetric unitary matrix to avoid degenerate neutrino masses, by performing the Takagi factorization X_ν can be written as [21, 33]

$$X_\nu = \Sigma_\nu \Sigma_\nu^T, \quad (3.10)$$

\widehat{X}_N	\widehat{X}_ν	R (NO)	R (IO)
diag(1, 1)	$\mathcal{D}(1, 1)$	$\begin{pmatrix} 0 & \cos \vartheta & \xi \sin \vartheta \\ 0 & -\sin \vartheta & \xi \cos \vartheta \end{pmatrix}$	$\begin{pmatrix} \cos \vartheta & \xi \sin \vartheta & 0 \\ -\sin \vartheta & \xi \cos \vartheta & 0 \end{pmatrix}$
diag(1, 1)	$\mathcal{D}(1, -1)$	\mathbf{X}	\mathbf{X}
diag(1, 1)	$\mathcal{D}(-1, 1)$	\mathbf{X}	\mathbf{X}
diag(1, 1)	$\mathcal{D}(-1, -1)$	\mathbf{X}	\mathbf{X}
diag(1, -1)	$\mathcal{D}(1, 1)$	\mathbf{X}	\mathbf{X}
diag(1, -1)	$\mathcal{D}(1, -1)$	$\pm \begin{pmatrix} 0 & \cosh \vartheta & i\xi \sinh \vartheta \\ 0 & -i \sinh \vartheta & \xi \cosh \vartheta \end{pmatrix}$	$\pm \begin{pmatrix} \cosh \vartheta & i\xi \sinh \vartheta & 0 \\ -i \sinh \vartheta & \xi \cosh \vartheta & 0 \end{pmatrix}$
diag(1, -1)	$\mathcal{D}(-1, 1)$	$\pm \begin{pmatrix} 0 & i \sinh \vartheta & -\xi \cosh \vartheta \\ 0 & \cosh \vartheta & i\xi \sinh \vartheta \end{pmatrix}$	$\pm \begin{pmatrix} i \sinh \vartheta & -\xi \cosh \vartheta & 0 \\ \cosh \vartheta & i\xi \sinh \vartheta & 0 \end{pmatrix}$
diag(1, -1)	$\mathcal{D}(-1, -1)$	\mathbf{X}	\mathbf{X}

Table 1: The explicit form of R -matrix for all possible independent values of \widehat{X}_N and \widehat{X}_ν , where ϑ is a real free parameter. The symbol “ \mathbf{X} ” denotes that the solution for R -matrix does not exist since it has to fulfill the equality of Eq. (2.6). The notation $\mathcal{D}(x, y)$ with $x, y = \pm 1$ refers to $\text{diag}(e^{i\alpha}, x, y)$ and $\text{diag}(x, y, e^{i\alpha})$ for NO and IO respectively.

where Σ_ν is a unitary matrix and it can be expressed in terms of the eigenvalues and eigenvectors of X_ν [33]. Thus the constraint on the neutrino diagonalization matrix U_ν in Eq. (3.5) can be simplified into

$$\Sigma_\nu^T U_\nu^* \widehat{X}_\nu^{-\frac{1}{2}} = \Sigma_\nu^\dagger U_\nu \widehat{X}_\nu^{\frac{1}{2}}. \quad (3.11)$$

The matrices on the two sides of this equation are unitary and complex conjugates of each other. Therefore the combination $\Sigma_\nu^\dagger U_\nu \widehat{X}_\nu^{\frac{1}{2}}$ is a generic real orthogonal matrix, and consequently the unitary transformation U_ν takes the form [33, 50, 51]

$$U_\nu = \Sigma_\nu O_{3 \times 3} \widehat{X}_\nu^{-\frac{1}{2}}, \quad (3.12)$$

where $O_{3 \times 3}$ is a three dimensional real orthogonal matrix, and it can be generally parameterized as

$$O_{3 \times 3}(\theta_1, \theta_2, \theta_3) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_1 & \sin \theta_1 \\ 0 & -\sin \theta_1 & \cos \theta_1 \end{pmatrix} \begin{pmatrix} \cos \theta_2 & 0 & \sin \theta_2 \\ 0 & 1 & 0 \\ -\sin \theta_2 & 0 & \cos \theta_2 \end{pmatrix} \begin{pmatrix} \cos \theta_3 & \sin \theta_3 & 0 \\ -\sin \theta_3 & \cos \theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (3.13)$$

where θ_i ($i = 1, 2, 3$) are real free parameters in the range of $[0, \pi)$. In our working basis (usually called leptogenesis basis) where the charged lepton mass matrix is diagonal, lepton flavor mixing completely arises from the neutrino sector, and therefore the lepton mixing matrix U coincides with U_ν . Hence we conclude that the mixing matrix and all mixing angles and CP phases would depend on three free continuous parameters $\theta_{1,2,3}$ if only one residual CP transformation is preserved in the neutrino sector. In order to facilitate the discussion of leptogenesis, we separate out the CP parity matrices \widehat{X}_N and \widehat{X}_ν and define the following three parameters

$$U' \equiv U \widehat{X}_\nu^{\frac{1}{2}}, \quad R' \equiv \widehat{X}_N^{-\frac{1}{2}} R \widehat{X}_\nu^{\frac{1}{2}}, \quad K_i \equiv (\widehat{X}_N)_{11} (\widehat{X}_\nu^{-1})_{ii}, \quad i = 1, 2, 3. \quad (3.14)$$

We see that R' is real and the parameter K_i is equal to $+1$, -1 or $\pm e^{-i\alpha}$. As a consequence, the flavored CP asymmetry ϵ_α can be expressed as

$$\epsilon_\alpha = -\frac{3M_1}{16\pi v^2} \frac{\Im(\sum_{ij} \sqrt{m_i m_j} m_j R'_{1i} R'_{1j} U'^*_{\alpha i} U'_{\alpha j} K_j)}{\sum_j m_j R'^2_{1j}}, \quad (3.15)$$

and the washout mass \tilde{m}_α is given by

$$\tilde{m}_\alpha = \left| \sum_i \sqrt{m_i} R'_{1i} U'_{\alpha i} \right|^2. \quad (3.16)$$

Taking into account that the lightest neutrino is massless in 2RHN model, we find ϵ_α and \tilde{m}_α can be written into a rather simple form

$$\text{NO} : \epsilon_\alpha = -\frac{3M_1}{16\pi v^2} W_{\text{NO}} I_{\text{NO}}^\alpha, \quad \tilde{m}_\alpha = \left| \sqrt{m_3} R'_{13} U'_{\alpha 3} + \sqrt{m_2} R'_{12} U'_{\alpha 2} \right|^2, \quad (3.17a)$$

$$\text{IO} : \epsilon_\alpha = -\frac{3M_1}{16\pi v^2} W_{\text{IO}} I_{\text{IO}}^\alpha, \quad \tilde{m}_\alpha = \left| \sqrt{m_2} R'_{12} U'_{\alpha 2} + \sqrt{m_1} R'_{11} U'_{\alpha 1} \right|^2, \quad (3.17b)$$

with

$$\begin{aligned} W_{\text{NO}} &= \frac{\sqrt{m_2 m_3} R'_{12} R'_{13} (m_3 K_3 - m_2 K_2)}{m_2 R_{12}'^2 + m_3 R_{13}'^2}, & I_{\text{NO}}^\alpha &= \Im(U'_{\alpha 3} U_{\alpha 2}'^*), \\ W_{\text{IO}} &= \frac{\sqrt{m_1 m_2} R'_{11} R'_{12} (m_2 K_2 - m_1 K_1)}{m_1 R_{11}'^2 + m_2 R_{12}'^2}, & I_{\text{IO}}^\alpha &= \Im(U'_{\alpha 2} U_{\alpha 1}'^*). \end{aligned} \quad (3.18)$$

The explicit expressions of W_{NO} and W_{IO} for the three viable forms of the R -matrix are shown in table 2. Notice that $W_{\text{NO,IO}}$ are fixed by the light neutrino masses $m_{2,3}$ and ϑ which parametrizes the R -matrix, and the bilinear invariants $I_{\text{NO,IO}}^\alpha$ depend on the low energy CP phases contained in the mixing matrix U . As a result, if the signal of CP violation was observed in future neutrino oscillation experiments or neutrinoless double decay experiments, a nonzero baryon asymmetry is expected to be generated through leptogenesis in this framework. In the following, we shall perform a general analysis of leptogenesis in the 2RHN model with a generic residual CP transformation, and the lepton mixing matrix can be parameterized as [52]

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \text{diag}(1, e^{i\frac{\phi}{2}}, 1), \quad (3.19)$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$, δ and ϕ and the Dirac type and Majorana type CP violating phases respectively. Note that there is only one Majorana CP phase ϕ in the presence of one massless light neutrino. Now we discuss the predictions for matter/antimatter asymmetry for each admissible R -matrix.

- R-1st

In this case, the CP asymmetry parameter ϵ_α for the NO case is given by

$$\begin{aligned} \epsilon_e &= \frac{3M_1}{16\pi v^2} W_{\text{NO}} s_{12}c_{13}s_{13} \sin\left(\delta + \frac{\phi}{2}\right), \\ \epsilon_\mu &= -\frac{3M_1}{16\pi v^2} W_{\text{NO}} c_{13}s_{23} \left[s_{12}s_{13}s_{23} \sin\left(\delta + \frac{\phi}{2}\right) - c_{12}c_{23} \sin \frac{\phi}{2} \right], \\ \epsilon_\tau &= -\frac{3M_1}{16\pi v^2} W_{\text{NO}} c_{13}c_{23} \left[s_{12}s_{13}c_{23} \sin\left(\delta + \frac{\phi}{2}\right) + c_{12}s_{23} \sin \frac{\phi}{2} \right], \end{aligned} \quad (3.20)$$

where the expression of W_{NO} has been listed in table 2. It is easy to check the identity $\epsilon_e + \epsilon_\mu + \epsilon_\tau = 0$ is fulfilled. Notice that the CP asymmetry ϵ_α is closely related to the lower energy CP phases. If both the Dirac phase δ and the Majorana phase ϕ are trivially zero, all the asymmetry

	Mass ordering	K_i	$(R'_{11}, R'_{12}, R'_{13})$	$W_{\text{NO}} (W_{\text{IO}})$
R-1st	NO	$K_2 = K_3 = 1$	$(0, \cos \vartheta, \xi \sin \vartheta)$	$\frac{\xi \sqrt{m_2 m_3} (m_3 - m_2) \sin 2\vartheta}{2(m_2 \cos^2 \vartheta + m_3 \sin^2 \vartheta)}$
	IO	$K_1 = K_2 = 1$	$(\cos \vartheta, \xi \sin \vartheta, 0)$	$\frac{\xi \sqrt{m_1 m_2} (m_2 - m_1) \sin 2\vartheta}{2(m_1 \cos^2 \vartheta + m_2 \sin^2 \vartheta)}$
R-2nd	NO	$K_2 = -K_3 = 1$	$\pm(0, \cosh \vartheta, -\xi \sinh \vartheta)$	$\frac{\xi \sqrt{m_2 m_3} (m_2 + m_3) \sinh 2\vartheta}{2(m_2 \cosh^2 \vartheta + m_3 \sinh^2 \vartheta)}$
	IO	$K_1 = -K_2 = 1$	$\pm(\cosh \vartheta, -\xi \sinh \vartheta, 0)$	$\frac{\xi \sqrt{m_1 m_2} (m_1 + m_2) \sinh 2\vartheta}{2(m_1 \cosh^2 \vartheta + m_2 \sinh^2 \vartheta)}$
R-3rd	NO	$-K_2 = K_3 = 1$	$\pm(0, -\sinh \vartheta, -\xi \cosh \vartheta)$	$\frac{\xi \sqrt{m_2 m_3} (m_2 + m_3) \sinh 2\vartheta}{2(m_2 \sinh^2 \vartheta + m_3 \cosh^2 \vartheta)}$
	IO	$-K_1 = K_2 = 1$	$\pm(-\sinh \vartheta, -\xi \cosh \vartheta, 0)$	$\frac{\xi \sqrt{m_1 m_2} (m_1 + m_2) \sinh 2\vartheta}{2(m_1 \sinh^2 \vartheta + m_2 \cosh^2 \vartheta)}$

Table 2: The parametrization of the first row of R' and the corresponding expressions of W_{NO} and W_{IO} for the three viable forms of the R -matrix.

parameters ϵ_e , ϵ_μ and ϵ_τ would be vanishing such that a nonzero baryon asymmetry can not be generated. The washout mass \tilde{m}_α for NO takes the form

$$\begin{aligned}
\tilde{m}_e &= \left| \sqrt{m_2} s_{12} c_{13} e^{\frac{i\phi}{2}} \cos \vartheta + \xi \sqrt{m_3} s_{13} e^{-i\delta} \sin \vartheta \right|^2, \\
\tilde{m}_\mu &= \left| \sqrt{m_2} \left(c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta} \right) e^{\frac{i\phi}{2}} \cos \vartheta + \xi \sqrt{m_3} c_{13} s_{23} \sin \vartheta \right|^2, \\
\tilde{m}_\tau &= \left| \sqrt{m_2} \left(c_{12} s_{23} + s_{12} s_{13} c_{23} e^{i\delta} \right) e^{\frac{i\phi}{2}} \cos \vartheta - \xi \sqrt{m_3} c_{13} c_{23} \sin \vartheta \right|^2.
\end{aligned} \tag{3.21}$$

In the same manner, we find ϵ_α for IO spectrum is

$$\begin{aligned}
\epsilon_e &= -\frac{3M_1}{16\pi v^2} W_{\text{IO}} c_{12} s_{12} c_{13}^2 \sin \frac{\phi}{2}, \\
\epsilon_\mu &= \frac{-3M_1}{16\pi v^2} W_{\text{IO}} \left[s_{13} c_{23} s_{23} (c_{12}^2 \sin(\delta - \frac{\phi}{2}) + s_{12}^2 \sin(\delta + \frac{\phi}{2})) - c_{12} s_{12} (c_{23}^2 - s_{13}^2 s_{23}^2) \sin \frac{\phi}{2} \right], \\
\epsilon_\tau &= \frac{3M_1}{16\pi v^2} W_{\text{IO}} \left[s_{13} c_{23} s_{23} (c_{12}^2 \sin(\delta - \frac{\phi}{2}) + s_{12}^2 \sin(\delta + \frac{\phi}{2})) + c_{12} s_{12} (s_{23}^2 - s_{13}^2 c_{23}^2) \sin \frac{\phi}{2} \right]
\end{aligned} \tag{3.22}$$

and for the washout mass \tilde{m}_α we get

$$\begin{aligned}
\tilde{m}_e &= c_{13}^2 \left| \sqrt{m_1} c_{12} \cos \vartheta + \xi \sqrt{m_2} s_{12} e^{\frac{i\phi}{2}} \sin \vartheta \right|^2, \\
\tilde{m}_\mu &= \left| \sqrt{m_1} (s_{12} c_{23} + c_{12} s_{13} s_{23} e^{i\delta}) \cos \vartheta - \xi \sqrt{m_2} (c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta}) e^{\frac{i\phi}{2}} \sin \vartheta \right|^2, \\
\tilde{m}_\tau &= \left| \sqrt{m_1} (s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i\delta}) \cos \vartheta - \xi \sqrt{m_2} (c_{12} s_{23} + s_{12} s_{13} c_{23} e^{i\delta}) e^{\frac{i\phi}{2}} \sin \vartheta \right|^2.
\end{aligned} \tag{3.23}$$

We see that both ϵ_α and \tilde{m}_α depend on the CP violating phases δ , ϕ and the free parameter ϑ . We display the contour regions for Y_B/Y_B^{obs} in the plane ϕ versus ϑ in figure 1, where the three mixing angles are taken to their best fit values [4] and the Dirac CP phase δ is either 0 or $-\pi/2$. The neutrino mass spectrum is NO and IO respectively in the first row and second row of this plot. We choose $\delta = 0$ in the left column and $\delta = -\pi/2$ in the right column. We find that the experimentally measured value of the baryon asymmetry can be accommodated in the case of NO, while Y_B is too small to account for its observed value for IO.

- R-2nd

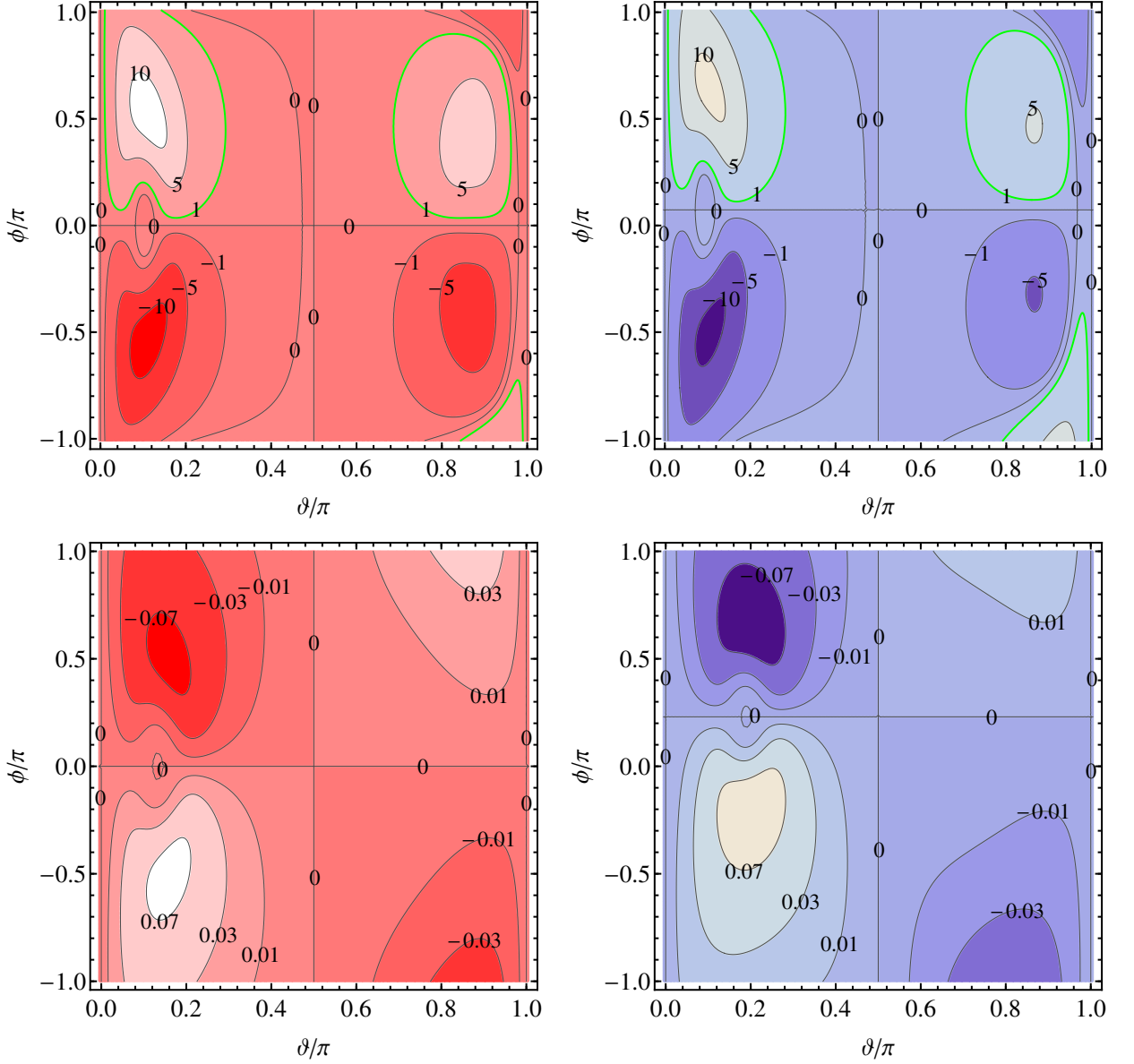


Figure 1: The contour plots of Y_B/Y_B^{obs} in the $\vartheta - \phi$ plane for the case of R-1st. Here we choose $M_1 = 5 \times 10^{11}$ GeV, so that only the tau Yukawa couplings are in equilibrium. The first row and the second row are for NO and IO spectrums respectively, and the Dirac CP phase δ is taken to be 0 on the left panels and $-\pi/2$ on the right panels. The neutrino oscillation parameters θ_{12} , θ_{13} , θ_{23} , δm^2 and Δm^2 are fixed at their best fit values [4]. The thick green curve represents the experimentally observed values of the baryon asymmetry $Y_B^{obs} = 8.66 \times 10^{-11}$ [53].

In this case, ϵ_α for NO is found to be,

$$\begin{aligned}
 \epsilon_e &= -\frac{3M_1}{16\pi v^2} W_{\text{NO}} s_{12} c_{13} s_{13} \cos\left(\delta + \frac{\phi}{2}\right), \\
 \epsilon_\mu &= -\frac{3M_1}{16\pi v^2} W_{\text{NO}} c_{13} s_{23} \left[c_{12} c_{23} \cos \frac{\phi}{2} - s_{12} s_{13} s_{23} \cos\left(\delta + \frac{\phi}{2}\right) \right], \\
 \epsilon_\tau &= \frac{3M_1}{16\pi v^2} W_{\text{NO}} c_{13} c_{23} \left[c_{12} s_{23} \cos \frac{\phi}{2} + s_{12} s_{13} c_{23} \cos\left(\delta + \frac{\phi}{2}\right) \right].
 \end{aligned} \tag{3.24}$$

The washout mass \tilde{m}_α is of the following form

$$\tilde{m}_e = \left| \sqrt{m_2} s_{12} c_{13} e^{\frac{i\phi}{2}} \cosh \vartheta - i \xi \sqrt{m_3} s_{13} e^{-i\delta} \sinh \vartheta \right|^2,$$

$$\begin{aligned}\tilde{m}_\mu &= \left| \sqrt{m_2} \left(c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} \right) e^{\frac{i\phi}{2}} \cosh \vartheta - i\xi\sqrt{m_3}c_{13}s_{23} \sinh \vartheta \right|^2, \\ \tilde{m}_\tau &= \left| \sqrt{m_2} \left(c_{12}s_{23} + s_{12}s_{13}c_{23}e^{i\delta} \right) e^{\frac{i\phi}{2}} \cosh \vartheta + i\xi\sqrt{m_3}c_{13}c_{23} \sinh \vartheta \right|^2.\end{aligned}\quad (3.25)$$

Similarly for IO mass spectrum, we have

$$\begin{aligned}\epsilon_e &= -\frac{3M_1}{16\pi v^2} W_{\text{IO}} c_{12}s_{12}c_{13}^2 \cos \frac{\phi}{2}, \\ \epsilon_\mu &= \frac{3M_1}{16\pi v^2} W_{\text{IO}} \left[s_{13}c_{23}s_{23}(c_{12}^2 \cos(\delta - \frac{\phi}{2}) - s_{12}^2 \cos(\delta + \frac{\phi}{2})) + c_{12}s_{12}(c_{23}^2 - s_{13}^2s_{23}^2) \cos \frac{\phi}{2} \right], \\ \epsilon_\tau &= -\frac{3M_1}{16\pi v^2} W_{\text{IO}} \left[s_{13}c_{23}s_{23}(c_{12}^2 \cos(\delta - \frac{\phi}{2}) - s_{12}^2 \cos(\delta + \frac{\phi}{2})) + c_{12}s_{12}(s_{13}^2c_{23}^2 - s_{23}^2) \cos \frac{\phi}{2} \right]\end{aligned}\quad (3.26)$$

and

$$\begin{aligned}\tilde{m}_e &= c_{13}^2 \left| \sqrt{m_1}c_{12} \cosh \vartheta - i\xi\sqrt{m_2}s_{12}e^{\frac{i\phi}{2}} \sinh \vartheta \right|^2, \\ \tilde{m}_\mu &= \left| \sqrt{m_1}(s_{12}c_{23} + c_{12}s_{13}s_{23}e^{i\delta}) \cosh \vartheta + i\xi\sqrt{m_2}(c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta})e^{\frac{i\phi}{2}} \sinh \vartheta \right|^2, \\ \tilde{m}_\tau &= \left| \sqrt{m_1}(s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta}) \cosh \vartheta + i\xi\sqrt{m_2}(c_{12}s_{23} + s_{12}s_{13}c_{23}e^{i\delta})e^{\frac{i\phi}{2}} \sinh \vartheta \right|^2\end{aligned}\quad (3.27)$$

The contour regions of Y_B/Y_B^{obs} are displayed in the plane ϕ versus ϑ in figure 2, where the lepton mixing angles and the mass-squared splittings are set to their best fit values and two representative values $\delta = 0, -\pi/2$ are considered. Note there are no phenomenologically viable points in the region of $|\vartheta| > 0.6\pi$. We see that the observed value of the baryon asymmetry in the Universe can be reproduced for both NO and IO. This result is different from that of the R-1st case.

- R-3rd

In the case of NO, we find the flavored CP asymmetry ϵ_α is

$$\begin{aligned}\epsilon_e &= \frac{3M_1}{16\pi v^2} W_{\text{NO}} s_{12}c_{13}s_{13} \cos(\delta + \frac{\phi}{2}), \\ \epsilon_\mu &= \frac{3M_1}{16\pi v^2} W_{\text{NO}} c_{13}s_{23} \left[c_{12}c_{23} \cos \frac{\phi}{2} - s_{12}s_{13}s_{23} \cos(\delta + \frac{\phi}{2}) \right], \\ \epsilon_\tau &= -\frac{3M_1}{16\pi v^2} W_{\text{NO}} c_{13}c_{23} \left[c_{12}s_{23} \cos \frac{\phi}{2} + s_{12}s_{13}c_{23} \cos(\delta + \frac{\phi}{2}) \right].\end{aligned}\quad (3.28)$$

It is easy to check the equality $\epsilon_2 \equiv \epsilon_e + \epsilon_\mu = -\epsilon_\tau$ is satisfied. The washout mass \tilde{m}_α takes the form

$$\begin{aligned}\tilde{m}_e &= \left| i\sqrt{m_2}s_{12}c_{13}e^{\frac{i\phi}{2}} \sinh \vartheta + \xi\sqrt{m_3}s_{13}e^{-i\delta} \cosh \vartheta \right|^2, \\ \tilde{m}_\mu &= \left| i\sqrt{m_2} \left(c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} \right) e^{\frac{i\phi}{2}} \sinh \vartheta + \xi\sqrt{m_3}c_{13}s_{23} \cosh \vartheta \right|^2, \\ \tilde{m}_\tau &= \left| i\sqrt{m_2} \left(c_{12}s_{23} + s_{12}s_{13}c_{23}e^{i\delta} \right) e^{\frac{i\phi}{2}} \sinh \vartheta - \xi\sqrt{m_3}c_{13}c_{23} \cosh \vartheta \right|^2.\end{aligned}\quad (3.29)$$

For the IO case, we can read out ϵ_α as

$$\begin{aligned}\epsilon_e &= \frac{3M_1}{16\pi v^2} W_{\text{IO}} c_{12}s_{12}c_{13}^2 \cos \frac{\phi}{2}, \\ \epsilon_\mu &= \frac{-3M_1}{16\pi v^2} W_{\text{IO}} \left(s_{13}c_{23}s_{23}(c_{12}^2 \cos(\delta - \frac{\phi}{2}) - s_{12}^2 \cos(\delta + \frac{\phi}{2})) + c_{12}s_{12}(c_{23}^2 - s_{13}^2s_{23}^2) \cos \frac{\phi}{2} \right), \\ \epsilon_\tau &= \frac{3M_1}{16\pi v^2} W_{\text{IO}} \left(s_{13}c_{23}s_{23}(c_{12}^2 \cos(\delta - \frac{\phi}{2}) - s_{12}^2 \cos(\delta + \frac{\phi}{2})) - c_{12}s_{12}(s_{23}^2 - s_{13}^2c_{23}^2) \cos \frac{\phi}{2} \right)\end{aligned}\quad (3.30)$$

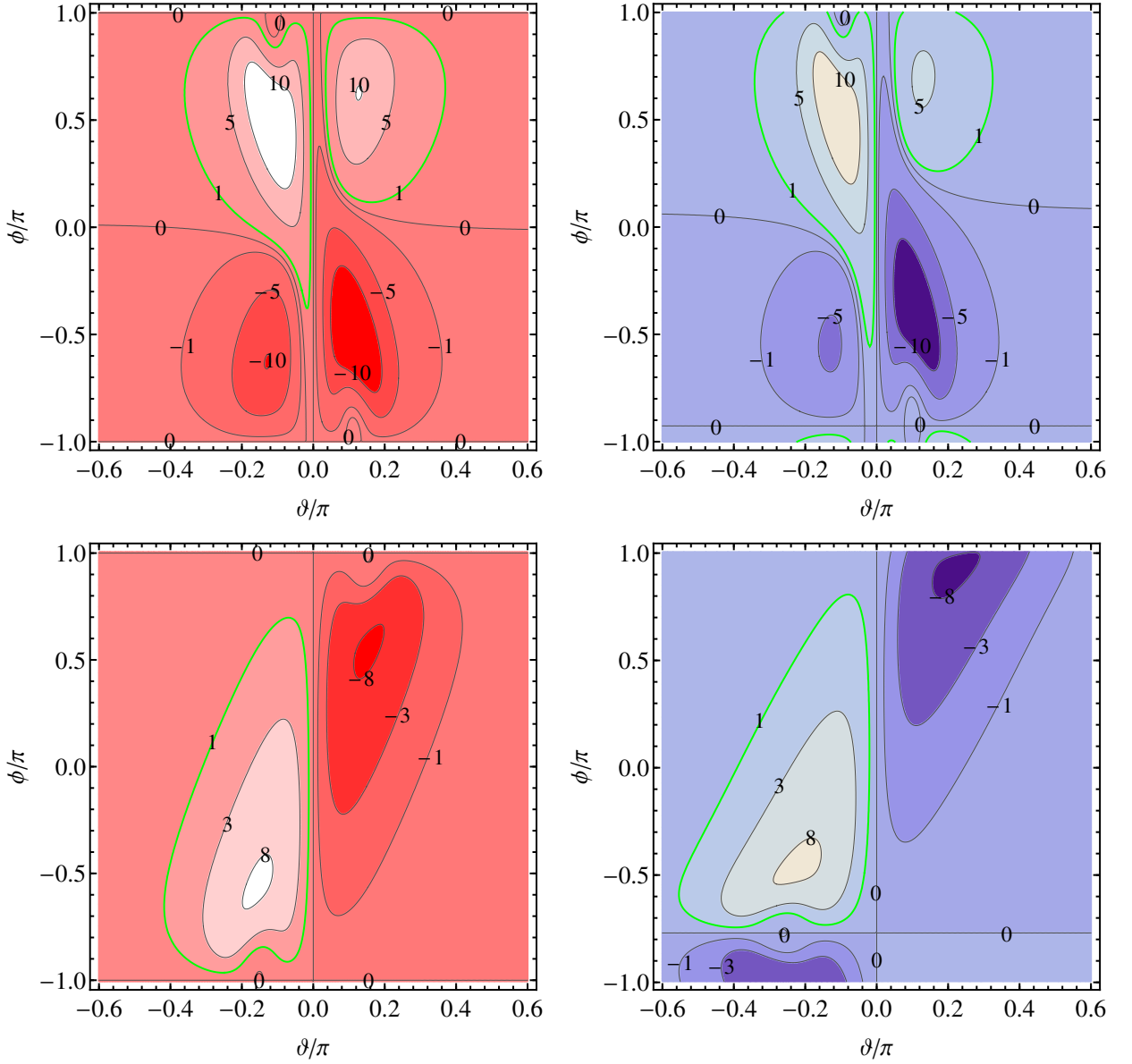


Figure 2: The contour plots of Y_B/Y_B^{obs} in the $\vartheta - \phi$ plane for the case of R-2nd. Here we choose $M_1 = 5 \times 10^{11}$ GeV, so that only the tau Yukawa couplings are in equilibrium. The first row and the second row are for NO and IO spectrums respectively, and the Dirac CP phase δ is taken to be 0 on the left panels and $-\pi/2$ on the right panels. The neutrino oscillation parameters θ_{12} , θ_{13} , θ_{23} , δm^2 and Δm^2 are fixed at their best fit values [4]. The thick green curve represents the experimentally observed values of the baryon asymmetry $Y_B^{obs} = 8.66 \times 10^{-11}$ [53].

Furthermore the washout mass \tilde{m}_α for IO turns out to be

$$\begin{aligned}
\tilde{m}_e &= c_{13}^2 \left| i\sqrt{m_1} c_{12} \sinh \vartheta + \xi \sqrt{m_2} s_{12} e^{\frac{i\phi}{2}} \cosh \vartheta \right|^2, \\
\tilde{m}_\mu &= \left| i\sqrt{m_1} (s_{12} c_{23} + c_{12} s_{13} s_{23} e^{i\delta}) \sinh \vartheta - \xi \sqrt{m_2} (c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta}) e^{\frac{i\phi}{2}} \cosh \vartheta \right|^2, \\
\tilde{m}_\tau &= \left| i\sqrt{m_1} (s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i\delta}) \sinh \vartheta - \xi \sqrt{m_2} (c_{12} s_{23} + s_{12} s_{13} c_{23} e^{i\delta}) e^{\frac{i\phi}{2}} \cosh \vartheta \right|^2. \quad (3.31)
\end{aligned}$$

We plot the contour regions of Y_B/Y_B^{obs} in the plane ϕ versus ϑ in figure 3, where we choose the lepton mixing angles as well as mass-squared differences to be their best fit values and $\delta = 0, -\pi/2$. Notice that the region of $|\vartheta| \geq 0.6\pi$ is not showed in figure 3 because the resulting

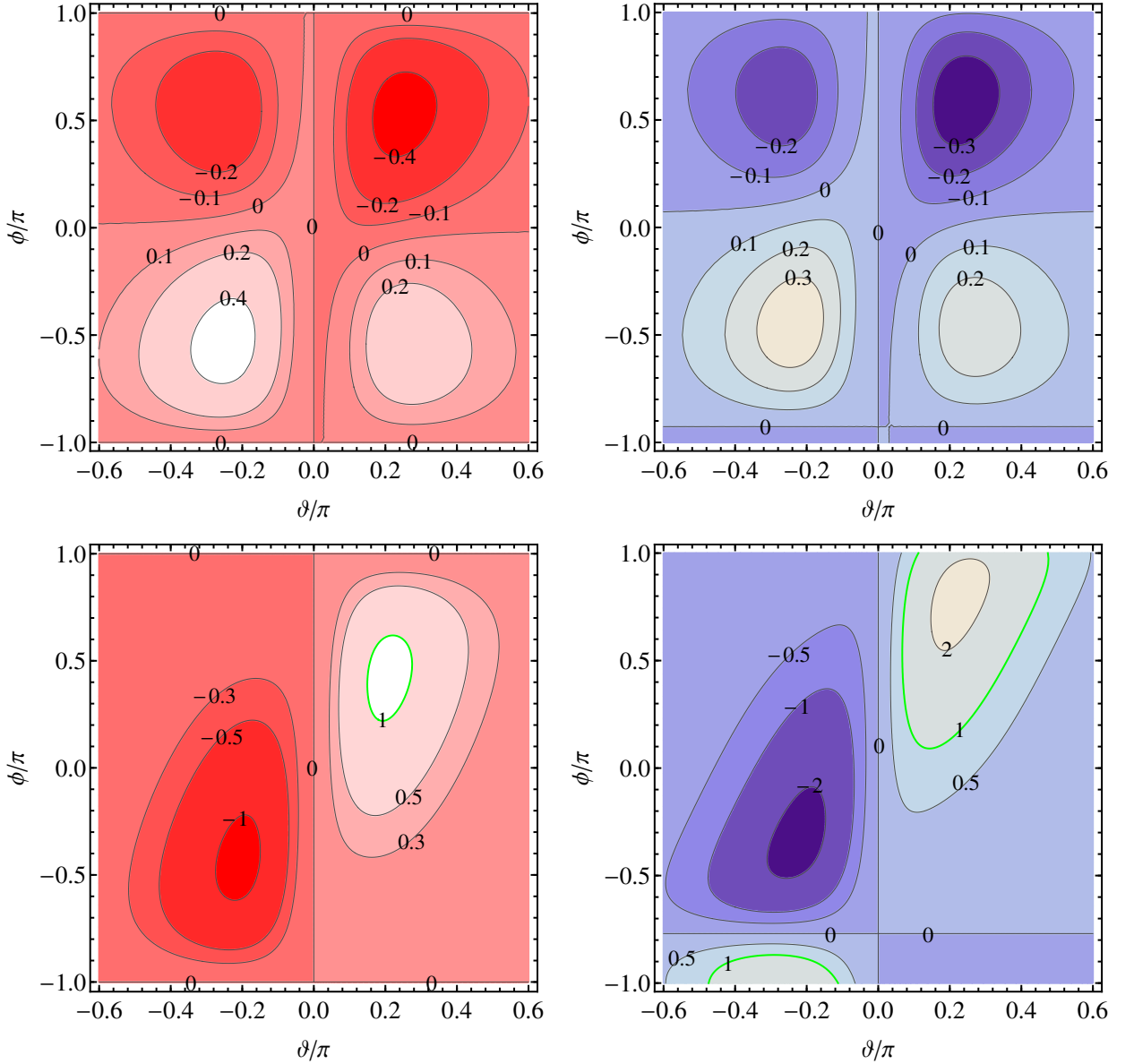


Figure 3: The contour plots of Y_B/Y_B^{obs} in the $\vartheta - \phi$ plane for the case of R-3rd. Here we choose $M_1 = 5 \times 10^{11}$ GeV, so that only the tau Yukawa couplings are in equilibrium. The first row and the second row are for NO and IO spectrums respectively, and the Dirac CP phase δ is taken to be 0 on the left panels and $-\pi/2$ on the right panels. The neutrino oscillation parameters θ_{12} , θ_{13} , θ_{23} , δm^2 and Δm^2 are fixed at their best fit values [4]. The thick green curve represents the experimentally observed values of the baryon asymmetry $Y_B^{obs} = 8.66 \times 10^{-11}$ [53].

predictions for Y_B/Y_B^{obs} are quite small. Moreover, we see that the realistic baryon asymmetry can be generated in the case of IO while Y_B is determined to be smaller than its observed value for NO.

4 Leptogenesis with two residual CP transformations or a cyclic residual flavor symmetry

In this section, we shall proceed to discuss the predictions for leptogenesis in the case that two residual CP transformations or a cyclic residual flavor symmetry is preserved by the seesaw Lagrangian in the 2RHN model.

4.1 Two residual CP transformations preserved

Following the same method as section 3, we investigate what we could learn if the parent CP symmetry at high energy scale is broken down to two residual CP transformations in the neutrino sectors. The lepton fields transform as

$$\begin{aligned} \nu_L &\xrightarrow{\text{CP}_1} iX_{\nu 1}\gamma_0 C\bar{\nu}_L^T, & N_R &\xrightarrow{\text{CP}_1} i\hat{X}_{N1}\gamma_0 C\bar{N}_R^T, \\ \nu_L &\xrightarrow{\text{CP}_2} iX_{\nu 2}\gamma_0 C\bar{\nu}_L^T, & N_R &\xrightarrow{\text{CP}_2} i\hat{X}_{N2}\gamma_0 C\bar{N}_R^T. \end{aligned} \quad (4.1)$$

with $X_{\nu 1} \neq X_{\nu 2}$ and $\hat{X}_{N1} \neq \hat{X}_{N2}$. The invariance of λ and M under the action of the above CP transformations $X_{\nu i}$ and \hat{X}_{Ni} implies

$$\hat{X}_{N1}^\dagger \lambda X_{\nu 1} = \lambda^*, \quad \hat{X}_{N1}^\dagger M \hat{X}_{N1}^* = M^*, \quad (4.2a)$$

$$\hat{X}_{N2}^\dagger \lambda X_{\nu 2} = \lambda^*, \quad \hat{X}_{N2}^\dagger M \hat{X}_{N2}^* = M^*. \quad (4.2b)$$

Notice that $-X_{\nu i}$, $-\hat{X}_{Ni}$ leads to the same constraints as $X_{\nu i}$, \hat{X}_{Ni} , hence they are identified as the same residual CP transformation. Because the RH neutrino fields N_{1R} and N_{2R} are assumed to be in the mass eigenstates, \hat{X}_{N1} and \hat{X}_{N2} must be diagonal with elements $+1$ or -1 . i.e.,

$$\hat{X}_{N1}, \hat{X}_{N2} = \text{diag}(\pm 1, \pm 1), \quad (4.3)$$

The light neutrino mass matrix m_ν is given by the seesaw relation. We can straightforwardly check that the residual CP transformations lead to the following two constraints on m_ν ,

$$X_{\nu 1}^T m_\nu X_{\nu 1} = m_\nu^*, \quad X_{\nu 2}^T m_\nu X_{\nu 2} = m_\nu^*. \quad (4.4)$$

This is exactly the condition that m_ν is invariant under the residual CP transformations $X_{\nu 1}$ and $X_{\nu 2}$. From Eq. (4.4) we can derive that the unitary transformation U_ν which diagonalizes m_ν should satisfy

$$U_\nu^\dagger X_{\nu 1} U_\nu^* = \hat{X}_{\nu 1}, \quad U_\nu^\dagger X_{\nu 2} U_\nu^* = \hat{X}_{\nu 2}, \quad (4.5)$$

with

$$\begin{aligned} \hat{X}_{\nu 1}, \hat{X}_{\nu 2} &= \text{diag}(e^{i\alpha_{1,2}}, \pm 1, \pm 1) && \text{for NO}, \\ \hat{X}_{\nu 1}, \hat{X}_{\nu 2} &= \text{diag}(\pm 1, \pm 1, e^{i\alpha_{1,2}}) && \text{for IO}, \end{aligned} \quad (4.6)$$

where α_1 and α_2 are arbitrary real parameters. Eq. (4.5) indicates that both residual CP transformations $X_{\nu 1}$ and $X_{\nu 2}$ must be symmetric unitary matrices. Using the symmetry properties of λ , M and U_ν shown in Eqs. (4.2a, 4.2b, 4.4), we find that the R -matrix is subject to the following constraints

$$\hat{X}_{N1} R^* \hat{X}_{\nu 1}^{-1} = R, \quad \hat{X}_{N2} R^* \hat{X}_{\nu 2}^{-1} = R, \quad (4.7)$$

which imply

$$R = \hat{X}_{N1} \hat{X}_{N2} R \hat{X}_{\nu 1} \hat{X}_{\nu 2}^{-1}. \quad (4.8)$$

Because the residual CP transformations $X_{\nu 1}$, \hat{X}_{N1} are distinct from $X_{\nu 2}$, \hat{X}_{N2} , the combinations $\hat{X}_{N1} \hat{X}_{N2}$ and $\hat{X}_{\nu 1} \hat{X}_{\nu 2}^{-1}$ should take the form²

$$\hat{X}_{N1} \hat{X}_{N2} = \text{diag}(1, -1), \quad \hat{X}_{\nu 1} \hat{X}_{\nu 2}^{-1} = P_\nu \text{diag}(e^{i(\alpha_1 - \alpha_2)}, 1, -1) P_\nu^T, \quad (4.9)$$

²The same results for the R -matrix would be obtained in the case of $\hat{X}_{N1} \hat{X}_{N2} = -\text{diag}(1, -1)$, $\hat{X}_{\nu 1} \hat{X}_{\nu 2}^{-1} = P_\nu \text{diag}(e^{i(\alpha_1 - \alpha_2)}, 1, -1) P_\nu^T$.

Mass Ordering	P_ν	$\hat{\theta}$	R
NO	P_{123}	$0, \pi$	$R = \begin{pmatrix} 0 & \pm 1 & 0 \\ 0 & 0 & \pm \xi \\ 0 & \mp 1 & 0 \end{pmatrix}$
	P_{132}	$\pm \frac{\pi}{2}$	$R = \begin{pmatrix} 0 & 0 & \pm \xi \\ 0 & \mp 1 & 0 \end{pmatrix}$
IO	P_{231}	$0, \pi$	$R = \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & \pm \xi & 0 \\ 0 & 0 & 1 \end{pmatrix}$
	P_{321}	$\pm \frac{\pi}{2}$	$R = \begin{pmatrix} 0 & \pm \xi & 0 \\ \mp 1 & 0 & 0 \end{pmatrix}$

Table 3: The explicit form of R -matrix for different possible values P_ν , where ξ is either $+1$ or -1 .

where P_ν is a permutation matrix with $P_\nu = P_{123}, P_{132}$ for NO and $P_\nu = P_{231}, P_{321}$ for IO. Here the six 3×3 permutation matrices are denoted as

$$\begin{aligned}
P_{123} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & P_{132} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, & P_{213} &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
P_{231} &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, & P_{312} &= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, & P_{321} &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.
\end{aligned} \tag{4.10}$$

Inserting Eq. (4.9) into Eq. (4.8) we obtain

$$RP_\nu = \text{diag}(1, -1)RP_\nu \text{diag}(e^{i(\alpha_1 - \alpha_2)}, 1, -1). \tag{4.11}$$

Consequently the (13) and (22) elements of the matrix RP_ν are vanishing. The explicit forms of the R -matrix for all possible values of P_ν are summarized in table 3. It is easy to check that all the flavored CP symmetry ϵ_α is vanishing, i.e.

$$\epsilon_e = \epsilon_\mu = \epsilon_\tau = 0. \tag{4.12}$$

As a result, a net baryon asymmetry can not be generated at leading order in this case, and moderate high order corrections are necessary in order to make leptogenesis viable. We would like to emphasize that this result is quite general and it is independent of the explicit form of the residual CP transformations $X_{\nu i}$ and \hat{X}_{Ni} .

4.2 A cyclic residual flavor symmetry preserved

In this section we shall proceed to discuss the implications of the residual flavor symmetry (without residual CP) for leptogenesis. We assume that the flavor symmetry group is broken down to a cyclic Z_n subgroup in the neutrino sector, where the subscript n denotes the order of the cyclic group. Under the action of the generator of the residual flavor symmetry Z_n , the neutrinos fields transform as

$$\nu_L \xrightarrow{Z_n} G_\nu \nu_L, \quad N_R \xrightarrow{Z_n} \hat{G}_N N_R, \tag{4.13}$$

where G_ν is a 3×3 unitary matrix with $G_\nu^n = 1_{3 \times 3}$ and $\hat{G}_N = \text{diag}(\pm 1, \pm 1)$ in our working basis. For this residual symmetry to hold, the Yukawa coupling λ and the RH neutrino mass matrix $M \equiv \text{diag}(M_1, M_2)$ have to fulfill

$$\hat{G}_N^\dagger \lambda G_\nu = \lambda, \quad \hat{G}_N^\dagger M \hat{G}_N^* = M. \tag{4.14}$$

\widehat{G}_N	\widehat{G}_ν	R (NO)	R (IO)
diag(1, -1)	$\mathcal{D}(1, 1)$	\mathbf{X}	\mathbf{X}
diag(1, -1)	$\mathcal{D}(1, -1)$	$\begin{pmatrix} 0 & \pm 1 & 0 \\ 0 & 0 & \pm \xi \end{pmatrix}$	$\begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & \pm \xi & 0 \end{pmatrix}$
diag(1, -1)	$\mathcal{D}(-1, 1)$	$\begin{pmatrix} 0 & 0 & \pm \xi \\ 0 & \mp 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \pm \xi & 0 \\ \mp 1 & 0 & 0 \end{pmatrix}$
diag(1, -1)	$\mathcal{D}(-1, -1)$	\mathbf{X}	\mathbf{X}
diag(-1, 1)	$\mathcal{D}(1, 1)$	\mathbf{X}	\mathbf{X}
diag(-1, 1)	$\mathcal{D}(1, -1)$	$\begin{pmatrix} 0 & 0 & \pm \xi \\ 0 & \mp 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \pm \xi & 0 \\ \mp 1 & 0 & 0 \end{pmatrix}$
diag(-1, 1)	$\mathcal{D}(-1, 1)$	$\begin{pmatrix} 0 & \pm 1 & 0 \\ 0 & 0 & \pm \xi \end{pmatrix}$	$\begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & \pm \xi & 0 \end{pmatrix}$
diag(1, 1)	$\mathcal{D}(-1, -1)$	\mathbf{X}	\mathbf{X}
-diag(1, 1)	$\mathcal{D}(1, 1)$	\mathbf{X}	\mathbf{X}
-diag(1, 1)	$\mathcal{D}(1, -1)$	\mathbf{X}	\mathbf{X}
-diag(1, 1)	$\mathcal{D}(-1, 1)$	\mathbf{X}	\mathbf{X}

Table 4: The explicit form of the R -matrix for different possible values of \widehat{G}_N and \widehat{G}_ν . The notation “ \mathbf{X} ” means that the solution for R -matrix does not exist, and $\mathcal{D}(x, y)$ with $x, y = \pm 1$ refers to $\text{diag}(e^{i\alpha}, x, y)$ and $\text{diag}(x, y, e^{i\alpha})$ for NO and IO respectively. Note that the residual flavor symmetry gives no constraint on the R -matrix for $\widehat{G}_N = -\text{diag}(1, 1)$ and $\widehat{G}_\nu = \mathcal{D}(-1, -1)$.

Subsequently we can check that the light neutrino mass matrix is invariant under the residual flavor symmetry

$$G_\nu^T m_\nu G_\nu = m_\nu. \quad (4.15)$$

From this condition we find that the neutrino diagonalization matrix U_ν can diagonalize the residual flavor symmetry transformation G_ν as well,

$$U_\nu^\dagger G_\nu U_\nu = \widehat{G}_\nu, \quad \text{with} \quad \widehat{G}_\nu = \begin{cases} \text{diag}(e^{i\alpha}, \pm 1, \pm 1) & \text{for NO} \\ \text{diag}(\pm 1, \pm 1, e^{i\alpha}) & \text{for IO} \end{cases}, \quad (4.16)$$

where $\alpha = 2\pi k/n$ with k coprime to n is a rational multiple of π . Notice that the maximal invariance group of the neutrino mass matrix is $U(1) \times Z_2 \times Z_2$ not a Klein group $Z_2 \times Z_2$ because one light neutrino mass is zero in this case. From Eq. (4.14) and Eq. (4.16), we can determine that the residual flavor symmetry gives rise to the following constraint on the R -matrix,

$$R = \widehat{G}_N R \widehat{G}_\nu. \quad (4.17)$$

The explicit forms of the R -matrix for all possible values of \widehat{G}_ν and \widehat{G}_N are listed in table 4. We see that there is only one nonzero element in each row of the R -matrix, consequently all the flavored CP asymmetries are zero

$$\epsilon_e = \epsilon_\mu = \epsilon_\tau = 0. \quad (4.18)$$

Hence the baryon asymmetry Y_B would be generally vanishing in the 2RHN model with a remnant Z_n flavor symmetry in the neutrino sector. In a concrete model, one could take into account the non-leading corrections arising from loop effects and higher dimensional operators to explain the correct size of matter/antimatter asymmetry [19].

5 Examples in $\Delta(6n^2)$ flavor symmetry and CP

In section 3, we have presented the general results for leptogenesis in the scenario that one residual CP transformation is preserved in the neutrino sector. In order to show concrete examples, we shall study the case that the single residual CP transformation arises from the breaking of the generalized CP symmetry compatible with the $\Delta(6n^2)$ flavor group.

$\Delta(6n^2)$ as flavor symmetry group and the resulting phenomenological consequence for lepton flavor mixing have been discussed in the literature [26–28, 54]. In the present work, we shall adopt the conventions and notations of Ref. [28] for the $\Delta(6n^2)$ group. The $\Delta(6n^2)$ group is isomorphic to $(Z_n \times Z_n) \rtimes S_3$ where the index n is a generic integer. The $\Delta(6n^2)$ group can be generated by four generators a, b, c and d which obey the following relations [28, 55]:

$$\begin{aligned} a^3 = b^2 = (ab)^2 = c^n = d^n = 1, \quad cd = dc, \quad aca^{-1} = c^{-1}d^{-1}, \\ ada^{-1} = c, \quad bcb^{-1} = d^{-1}, \quad bdb^{-1} = c^{-1}. \end{aligned} \quad (5.1)$$

The $\Delta(6n^2)$ group has $6n^2$ elements which can be expressed as

$$g = a^\alpha b^\beta c^\gamma d^\delta, \quad \alpha = 0, 1, 2, \quad \beta = 0, 1, \quad \gamma, \delta = 0, 1, \dots, n-1. \quad (5.2)$$

The group $\Delta(6n^2)$ has one-dimensional, two-dimensional, three-dimensional and six-dimensional irreducible representations [28, 55]. It has been shown that $\Delta(6n^2)$ has $2(n-1)$ three-dimensional irreducible representations denoted by $\mathbf{3}_{k,l}$ in which the explicit form of the four generators can be chosen as

$$\mathbf{3}_{k,l} : a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad b = (-1)^k \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad c = \begin{pmatrix} \eta^l & 0 & 0 \\ 0 & \eta^{-l} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \eta^l & 0 \\ 0 & 0 & \eta^{-l} \end{pmatrix}, \quad (5.3)$$

where $\eta \equiv e^{2\pi i/n}$, $k = 1, 2$ and $l = 1, 2, \dots, n-1$. In the following, without loss of generality we shall embed the three generations of left-handed lepton doublets into the faithful triplet $\mathbf{3}_{1,1}$ which is denoted by $\mathbf{3}$ for simplicity, while the two right-handed neutrinos are assumed to transform as a doublet of $\Delta(6n^2)$. As has been shown in Ref. [28], the most general CP transformation consistent with the $\Delta(6n^2)$ flavor symmetry is of the same form as the flavor symmetry transformation in the basis of Eq. (5.3), i.e.

$$X_{\mathbf{r}} = \rho_{\mathbf{r}}(g), \quad g \in \Delta(6n^2), \quad (5.4)$$

where $\rho_{\mathbf{r}}(g)$ denotes the representation matrix of the element g in the irreducible representation \mathbf{r} of the $\Delta(6n^2)$ group. Moreover, we assume that the $\Delta(6n^2)$ flavor symmetry is broken down to an abelian subgroup G_l in the charged lepton sector and G_l is capable of distinguishing among the three generations of the charged leptons. As a result, the charged lepton mass matrix is invariant under the action of the generator g_l of G_l ,

$$\rho_{\mathbf{3}}^\dagger(g_l) m_l^\dagger m_l \rho_{\mathbf{3}}(g_l) = m_l^\dagger m_l, \quad (5.5)$$

where the charged lepton mass matrix m_l is given in the right-left basis. The matrix $\rho_{\mathbf{3}}(g_l)$ can be diagonalized by a unitary transformation U_l ,

$$U_l^\dagger \rho_{\mathbf{3}}(g_l) U_l = \rho_{\mathbf{3}}^{\text{diag}}(g_l). \quad (5.6)$$

Then Eq. (5.5) implies that U_l also diagonalizes the charged lepton mass matrix $m_l^\dagger m_l$. Notice that U_l is uniquely determined up to permutations and phases of their column vectors. All possible residual subgroup G_l and the corresponding diagonalization matrices U_l are summarized in table 5, where G_l is assumed to be generated by a single generator. If we further take into account the

case that G_l is a product of several cyclic groups, the constraints on the parameters s and t in table 5 would be removed, yet no new additional form of U_l is generated [56]. In the neutrino sector, a single remnant CP transformation X_ν is preserved by the neutrino mass matrix such that the neutrino mixing matrix U_ν is of the form of Eq. (3.12), as shown in section 3. Hence the lepton mixing matrix is determined to be given by

$$U = P_l U_l^\dagger \Sigma_\nu O_{3 \times 3} \hat{X}_\nu^{-\frac{1}{2}}, \quad (5.7)$$

where P_l is a generic 3×3 permutation matrix since the charged lepton masses can not be predicted in this approach. One can straightforwardly check that two pairs of subgroups $\{G_l, X_\nu\}$ and $\{G'_l, X'_\nu\}$ would yield the same results for the PMNS mixing matrix [32], if they are related by a similarity transformation Ω ,

$$\rho_3(g'_l) = \Omega \rho_3(g_l) \Omega^\dagger, \quad X'_\nu = \Omega X_\nu \Omega^T, \quad (5.8)$$

where g_l and g'_l denote the generator of G_l and G'_l respectively. Moreover generally, we denote the mixing matrices predicted by two generic residual symmetries $\{G_l, X_\nu\}$ and $\{G'_l, X'_\nu\}$ as

$$U = P_l U_l^\dagger \Sigma_\nu O_{3 \times 3} \hat{X}_\nu^{-\frac{1}{2}}, \quad U' = P'_l U'^\dagger_l \Sigma'_\nu O'_{3 \times 3} \hat{X}'_\nu^{-\frac{1}{2}}, \quad (5.9)$$

The condition under which U and U' essentially lead to the same mixing pattern is found to be [32]

$$\Sigma \Sigma^T = Q_L P_L \Sigma' \Sigma'^T P_L^T Q_L, \quad (5.10)$$

where $\Sigma \equiv U_l^\dagger \Sigma_\nu$, $\Sigma' \equiv U'^\dagger_l \Sigma'_\nu$, $P_L \equiv P_l^T P'_l$, and Q_L is a diagonal phase matrix. As stated above, we assume that the concerned $\Delta(6n^2)$ flavor group and CP symmetry are broken to an abelian subgroup in the charged lepton sector and to a single remnant CP transformation X_ν in the neutrino sector. Thus X_ν has to be a symmetric unitary matrix and it can be

$$X_\nu = \rho_3(c^x d^y), \quad \rho_3(bc^x d^{-x}), \quad \rho_3(abc^x d^{2x}), \quad \rho_3(a^2 bc^{2x} d^x), \quad x, y = 0, 1, \dots, n-1, \quad (5.11)$$

which are related with each other by similarity transformation as follows

$$\begin{aligned} \rho_3(b) \rho_3(bc^x d^{-x}) \rho_3^T(b) &= \rho_3(bc^x d^{-x}), \\ \rho_3(a^2) \rho_3(bc^x d^{-x}) \rho_3^T(a^2) &= \rho_3(abc^x d^{2x}), \\ \rho_3(ad^{2x}) \rho_3(bc^x d^{-x}) \rho_3^T(ad^{2x}) &= \rho_3(a^2 bc^{2x} d^x). \end{aligned} \quad (5.12)$$

Hence it is sufficient to consider the choices of $X_\nu = \rho_3(c^x d^y)$ and $X_\nu = \rho_3(bc^x d^{-x})$ with $x, y = 0, 1, \dots, n-1$. The corresponding Takagi factorization matrix can be read out as,

$$X_\nu = \rho_3(c^x d^y), \quad \Sigma_\nu = \text{diag}(e^{\frac{x\pi i}{n}}, e^{\frac{(y-x)\pi i}{n}}, e^{-\frac{y\pi i}{n}}), \quad (5.13)$$

$$X_\nu = \rho_3(bc^x d^{-x}), \quad \Sigma_\nu = \begin{pmatrix} 0 & -ie^{\frac{x\pi i}{n}} & e^{\frac{x\pi i}{n}} \\ \sqrt{2}e^{-\frac{2x\pi i}{n}} & 0 & 0 \\ 0 & ie^{\frac{x\pi i}{n}} & e^{\frac{x\pi i}{n}} \end{pmatrix}. \quad (5.14)$$

Furthermore taking into account the following conjugate relations

$$\begin{aligned} b(abc^s d^t) b^{-1} &= a^2 bc^{-t} d^{-s}, \\ \rho_3(b) \rho_3(c^x d^y) \rho_3^T(b) &= \rho_3(c^{-y} d^{-x}), \\ \rho_3(b) \rho_3(bc^x d^{-x}) \rho_3^T(b) &= \rho_3(bc^x d^{-x}), \end{aligned} \quad (5.15)$$

we only need to consider eight possible remnant symmetries constituted by $G_l = \langle c^s d^t \rangle$, $\langle bc^s d^t \rangle$, $\langle ac^s d^t \rangle$, $\langle abc^s d^t \rangle$ and $X_\nu = \rho_3(c^x d^y)$, $X_\nu = \rho_3(bc^x d^{-x})$. In the following, we shall investigate the predictions for lepton flavor mixing and matter-antimatter asymmetry via leptogenesis in each possible case.

G_l	U_l	Constraints
$\langle c^s d^t \rangle$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$s + t \neq 0 \bmod(n)$ $s - 2t \neq 0 \bmod(n)$ $t - 2s \neq 0 \bmod(n)$
$\langle bc^s d^t \rangle$	$\frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\pi \frac{s+t}{2n}} & 0 & e^{-i\pi \frac{s+t}{2n}} \\ 0 & \sqrt{2} & 0 \\ -e^{i\pi \frac{s+t}{2n}} & 0 & e^{i\pi \frac{s+t}{2n}} \end{pmatrix}$	$s - t \neq 0, \frac{n}{3}, \frac{2n}{3} \bmod(n)$
$\langle ac^s d^t \rangle$	$\frac{1}{\sqrt{3}} \begin{pmatrix} e^{-2i\pi \frac{s}{n}} & \omega^2 e^{-2i\pi \frac{s}{n}} & \omega e^{-2i\pi \frac{s}{n}} \\ e^{-2i\pi \frac{t}{n}} & \omega e^{-2i\pi \frac{t}{n}} & \omega^2 e^{-2i\pi \frac{t}{n}} \\ 1 & 1 & 1 \end{pmatrix}$	—
$\langle a^2 c^s d^t \rangle$	$\frac{1}{\sqrt{3}} \begin{pmatrix} e^{-2i\pi \frac{t}{n}} & \omega^2 e^{-2i\pi \frac{t}{n}} & \omega e^{-2i\pi \frac{t}{n}} \\ e^{2i\pi \frac{s-t}{n}} & \omega e^{2i\pi \frac{s-t}{n}} & \omega^2 e^{2i\pi \frac{s-t}{n}} \\ 1 & 1 & 1 \end{pmatrix}$	—
$\langle abc^s d^t \rangle$	$\frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\pi \frac{t-2s}{2n}} & e^{i\pi \frac{t-2s}{2n}} & 0 \\ -e^{-i\pi \frac{t-2s}{2n}} & e^{-i\pi \frac{t-2s}{2n}} & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$	$t \neq 0, \frac{n}{3}, \frac{2n}{3}$
$\langle a^2 bc^s d^t \rangle$	$\frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & e^{i\pi \frac{s-2t}{2n}} & e^{i\pi \frac{s-2t}{2n}} \\ 0 & -e^{-i\pi \frac{s-2t}{2n}} & e^{-i\pi \frac{s-2t}{2n}} \end{pmatrix}$	$s \neq 0, \frac{n}{3}, \frac{2n}{3}$

Table 5: The unitary transformation U_l for the possible remnant subgroup G_l . Here the notation $\langle g \rangle$ denotes a group generated by the element g . The allowed values of the parameters s and t are $s, t = 0, 1, \dots, n-1$ and $\omega = e^{2\pi i/3}$ is the cube root of unit. Note that the identity $(ac^t d^{t-s})^2 = a^2 c^s d^t$ is fulfilled, consequently the unitary matrix U_l for $G_l = \langle a^2 c^s d^t \rangle$ can be obtained from that corresponding one of $G_l = \langle ac^s d^t \rangle$ through the replacement $s \rightarrow t$ and $t \rightarrow t - s$. The constraints on the parameters s and t are to eliminate the degeneracy among the eigenvalues of the generator of G_l , and they can be completely relaxed by extending G_l to be the direct product of several cyclic groups [56].

(I) $G_l = \langle c^s d^t \rangle$, $X_\nu = \rho_3(c^x d^y)$

From table 5 and Eq. (5.13) we find that the lepton mixing matrix is given by

$$U_I = P_l O_{3 \times 3}(\theta_1, \theta_2, \theta_3) \hat{X}_\nu^{-\frac{1}{2}}. \quad (5.16)$$

The permutation matrix P_l can be absorbed into the orthogonal matrix $O_{3 \times 3}$, hence we can choose $P_l = P_{123} = 1_{3 \times 3}$ without loss of generality. Thus the three lepton mixing angles read

$$\sin^2 \theta_{12} = \sin^2 \theta_3, \quad \sin^2 \theta_{13} = \sin^2 \theta_2, \quad \sin^2 \theta_{23} = \sin^2 \theta_1 \quad (5.17)$$

and the Jarlskog invariant J_{CP} is vanishing

$$J_{CP} = 0, \quad (5.18)$$

where J_{CP} is defined as [57]

$$J_{CP} = \Im(U_{11} U_{33} U_{13}^* U_{31}^*) = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \cos \theta_{13} \sin \delta. \quad (5.19)$$

Consequently the Dirac CP phase δ is either 0 or π . Moreover, we can easily check that both the rephase invariants I_{NO}^α , I_{IO}^α and the CP asymmetry ϵ_α in leptogenesis are vanishing as well,

$$I_{\text{NO}}^\alpha = I_{\text{IO}}^\alpha = \epsilon_\alpha = 0. \quad (5.20)$$

Therefore a net baryon asymmetry can not be generated in this case, and moderate subleading corrections are necessary in order to make the leptogenesis viable.

(II) $G_l = \langle c^s d^t \rangle$, $X_\nu = \rho_3(bc^x d^{-x})$

In this case, the PMNS mixing matrix is determined to be of the form

$$\begin{aligned} U_{II} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 1 \\ \sqrt{2} & 0 & 0 \\ 0 & i & 1 \end{pmatrix} O_{3 \times 3}(\theta_1, \theta_2, \theta_3) \hat{X}_\nu^{-\frac{1}{2}} \\ &= \text{diag}(e^{-i\theta_1}, 1, e^{i\theta_1}) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 1 \\ \sqrt{2} & 0 & 0 \\ 0 & i & 1 \end{pmatrix} O_{3 \times 3}(0, \theta_2, \theta_3) \hat{X}_\nu^{-\frac{1}{2}} \end{aligned} \quad (5.21)$$

up to possible permutations of rows. The diagonal phase matrix $\text{diag}(e^{-i\theta_1}, 1, e^{i\theta_1})$ can be absorbed into the charged lepton fields. Moreover, it is easy to check that the following identity is fulfilled

$$P_{321} U_{II}(\theta_1, \theta_2, \theta_3) = U_{II}(-\theta_1, \theta_2, -\theta_3) \text{diag}(1, -1, 1). \quad (5.22)$$

Consequently the six possible row permutations lead to three independent mixing patterns,

$$U_{II,1} = U_{II}, \quad U_{II,2} = P_{132} U_{II}, \quad U_{II,3} = P_{213} U_{II}. \quad (5.23)$$

We find that $U_{II,1}$ and $U_{II,2}$ predict $\tan \theta_{13} = \cos \theta_{23}$ and $\tan \theta_{13} = \sin \theta_{23}$ respectively such that the experimental data [4] of the mixing angles θ_{13} and θ_{23} can not be accommodated simultaneously. For the mixing matrix $U_{II,3}$, the lepton mixing parameters are given by

$$\begin{aligned} \sin^2 \theta_{13} &= \sin^2 \theta_2, \quad \sin^2 \theta_{12} = \sin^2 \theta_3, \quad \sin^2 \theta_{23} = \frac{1}{2}, \\ J_{CP} &= \frac{1}{8} \cos \theta_2 \sin 2\theta_2 \sin 2\theta_3, \quad |\sin \delta| = 1. \end{aligned} \quad (5.24)$$

We see that both Dirac CP phase δ and the atmospheric mixing angle θ_{23} are maximal while the values of θ_{12} and θ_{13} are not constrained. The best fitting values of $(\sin^2 \theta_{13})^{\text{bf}} = 0.0234$ ($(\sin^2 \theta_{13})^{\text{bf}} = 0.0240$) and $(\sin^2 \theta_{12})^{\text{bf}} = 0.308$ [4] for NO (IO) can be reproduced for certain values of the parameters $\theta_{1,2,3}$, as shown in table 6. Since the lepton mixing angles in Eq. (5.24) are invariant under the transformations $(\theta_2, \theta_3) \rightarrow (\pi - \theta_2, \theta_3)$, $(\theta_2, \theta_3) \rightarrow (\theta_2, \pi - \theta_3)$ and $(\theta_2, \theta_3) \rightarrow (\pi - \theta_2, \pi - \theta_3)$, four best fitting values for $\theta_{2,3}$ can be found. Note that next generation neutrino experiments [58, 59] is capable of testing the predictions for maximal δ and θ_{23} . Furthermore, we find that the rephasing bilinear invariants take the form

$$\begin{aligned} I_{\text{NO}}^e &= 0, \quad I_{\text{NO}}^\mu = -I_{\text{NO}}^\tau = \frac{1}{2} \cos \theta_2 \cos \theta_3, \\ I_{\text{IO}}^e &= 0, \quad I_{\text{IO}}^\mu = -I_{\text{IO}}^\tau = \frac{1}{2} \sin \theta_2. \end{aligned} \quad (5.25)$$

Hence only the muon and tau flavored asymmetries in heavy neutrino decay contribute to the leptogenesis. We plot the numerical results of the baryon asymmetry Y_B with respect to the free parameter ϑ in figure 4, where the parameters $\theta_{1,2,3}$ are set to their best fit values. Obviously the observed mater-antimatter asymmetry in the universe can be obtained for particular values of ϑ except the cases of R-3rd with NO spectrum and R-1st of IO. This conclusion is consistent with the general results of section 3.

	ϱ_i		θ_1^{bf}/π	θ_2^{bf}/π	θ_3^{bf}/π	χ^2_{min}	$\sin^2 \theta_{13}$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{23}$	$ \sin \delta $	$ \sin \phi $														
$U_{II,3}$	—	NO	—	0.049	0.187	3.645	0.0234	0.308	0.5	1	0														
					0.813																				
				0.951	0.187																				
		IO	—	0.050	0.813	0.105	0.024																		
					0.187																				
				0.950	0.813																				
$U_{III,1}$	$\varrho_1 = \frac{\pi}{6}$	NO	0.322	0.155	0.614	27.205	0.0295	0.308	0.577	0.985	0.253														
					0.977																				
			0.678	0.845	0.386																				
		IO	0.340	0.143	0.023	2.143	0.0251		0.641	0.983	0.789														
					0.606																				
			0.660	0.857	0.968																				
$U_{III,2}$	$\varrho_1 = \frac{\pi}{6}$	NO	0.329	0.150	0.394	7.674	0.0278	0.308	0.398	0.984	0.829														
					0.974																				
			0.671	0.850	0.389																				
		IO	0.331	0.149	0.026	7.281	0.0274		0.393	0.984	0.773														
					0.610																				
			0.669	0.851	0.973																				
$U_{III,3}$	$\varrho_1 = \frac{\pi}{3}$	NO	0.049	0.040	0.390	0	0.0234	0.308	0.437	0.873	0.852														
					0.027																				
			0.306																						
			0.681																						
			0.319																						
			0.694																						
		IO	0.050	0.028	0.035		0.024		0.455	0.870	0														
					0.409																				
			0.591																						
			0.965																						
			0.308																						
			0.683																						
$U_{V,1}$	$\varrho_3 = 0, \varrho_4 = \frac{\pi}{2}$	NO	0.454	0.694	0.317	3.327	0.0233	0.339	0.433	0.931	0.072														
					0.808																				
		IO	0.465	0.692	0.683							3.599	0.0238	0.340	0.449	0.961	0								
					0.809																				
		$U_{V,1}$	$\varrho_3 = 0, \varrho_4 = \frac{\pi}{3}$	NO	0.448													0.684	0.017	0	0.0234	0.308	0.437	0.899	0.929
																			0.746						
0.465	0.798				0.003	0.703	0.848																		
IO	0.463			0.684	0.740	0.024	0.455	0.984	0.892																
					0.004			0.097	0.999																
	0.475			0.803	0.736			0.936	0.826																
U_{VI}	$\varrho_5 = \frac{\pi}{2}$	NO	0.530	0.623	0.854	0.0235	0.323	0.448	0.894	0.016															
				0.925																					
		IO	0.508	0.731							0.349	0.0240	0.317	0.487	0.994	0									
				0.995																					
		$U_{VIII,1}$	—	NO													1	0.862	0.106	18.549	0.0244	0.308	0.578	0.667	0.580
																			0.894						
0	0.138				0.106																				
IO	1			0.861	0.894	0.794	0.024	0.579	0.668	0.616															
					0						0.139	0.106													
	0.894																								
$U_{VIII,2}$	—	NO	1	0.861	0.106	0.537	0.0236	0.308	0.420	0.389	0.616														
					0.894																				
			0	0.139	0.106																				
		IO	0	0.138	0.894	1.182	0.0242		0.422	0.667	0.615														
					0.106																				
			1	0.862	0.894																				

Table 6: Results of the χ^2 analysis for some representative mixing patterns which arise from the breaking of the $\Delta(6n^2)$ flavor group and CP to an abelian subgroup in the charged lepton sector and a single remnant CP transformation in the neutrino sector. The χ^2 function has a global minimum χ^2_{min} at the best fit values θ_1^{bf} , θ_2^{bf} and θ_3^{bf} for θ_1 , θ_2 and θ_3 . We display the values of the mixing angles as well as $|\sin \delta|$ and $|\sin \phi|$ at the given $\theta_{1,2,3}^{\text{bf}}$.

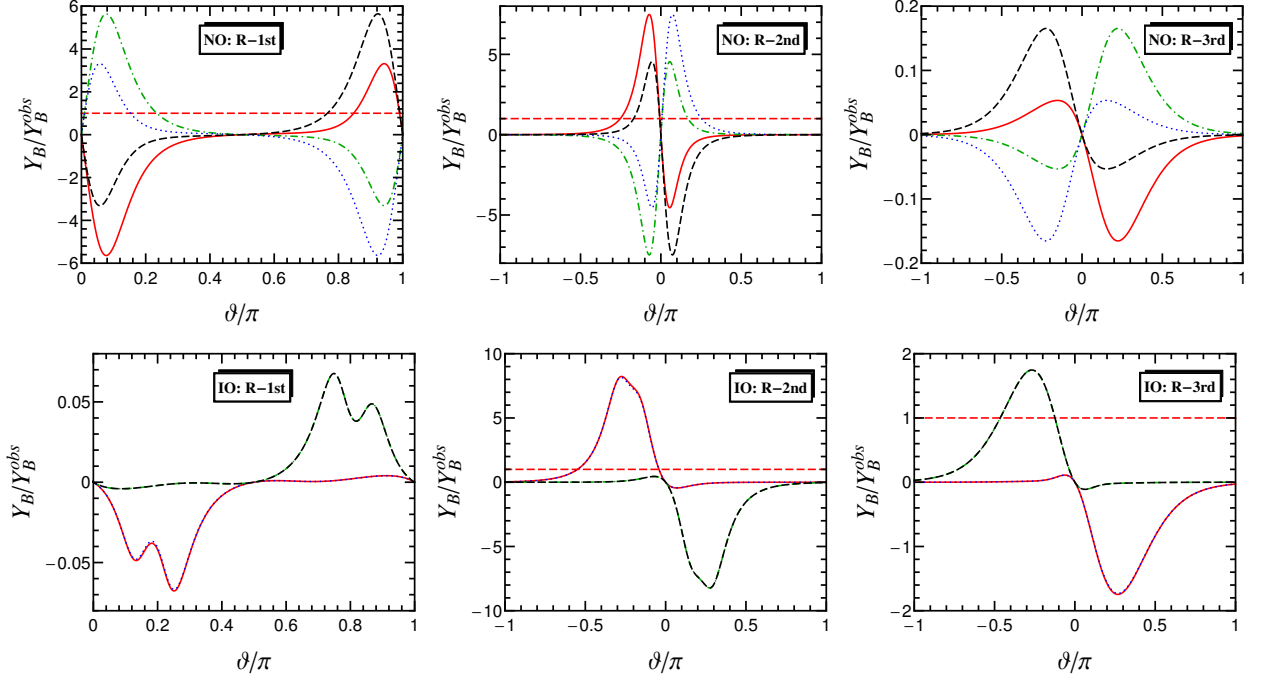


Figure 4: Y_B/Y_B^{obs} as a function of the parameter ϑ in case II, where we choose the RH neutrino mass $M_1 = 5 \times 10^{11}$ GeV. The red solid, green dash-dotted, blue dotted and black dashed lines correspond to the four best fitting points shown in table 6 one by one. The horizontal red dashed line represents the experimental measured value Y_B^{obs} . The neutrino mass spectrum is NO and IO in the first row and the second row respectively. The panels in the left, middle and right columns are for the three admissible forms of the R -matrix such as R-1st, R-2nd and R-3rd respectively.

(III) $G_l = \langle bc^s d^t \rangle$, $X_\nu = \rho_3(c^x d^y)$

Using table 5 and Eq. (5.13), we find that the lepton mixing matrix up to possible permutations of rows is fixed to be

$$U_{III} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -e^{i\varrho_1} \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & e^{i\varrho_1} \end{pmatrix} O_{3 \times 3}(\theta_1, \theta_2, \theta_3) \hat{X}_\nu^{-\frac{1}{2}}, \quad (5.26)$$

with

$$\varrho_1 = -\frac{(s+t+x+y)}{n}\pi, \quad (5.27)$$

which can take the following values

$$\varrho_1 \pmod{2\pi} = 0, \frac{1}{n}\pi, \frac{2}{n}\pi, \dots, \frac{2n-1}{n}\pi. \quad (5.28)$$

We can easily check that the mixing matrix U_{III} has the properties

$$\begin{aligned} U_{III}(\varrho_1 + \pi, \theta_1, \theta_2, \theta_3) &= U_{III}(\varrho_1, -\theta_1, -\theta_2, \theta_3) \text{diag}(1, 1, -1), \\ U_{III}(\pi - \varrho_1, \theta_1, \theta_2, \theta_3) &= \text{diag}(-e^{-i\varrho_1}, 1, e^{-i\varrho_1}) U_{III}(\varrho_1, \theta'_1, \theta'_2, \theta'_3) \text{diag}(1, 1, -1), \end{aligned} \quad (5.29)$$

where the parameters $\theta'_{1,2,3}$ fulfill $O_{3 \times 3}(\theta'_1, \theta'_2, \theta'_3) = P_{321} O_{3 \times 3}(-\theta_1, -\theta_2, \theta_3)$. As a consequence, the fundamental interval of the parameter ϱ_1 can be chosen to be $0 \leq \varrho_1 \leq \frac{\pi}{2}$. The mixing pattern arising from the multiplication of the permutation matrix P_{321} from the left-hand side, is related to U_{III} through shifts of the continuous parameters $\theta_{1,2,3}$ and redefining \hat{X}_ν as follow,

$$P_{321} U_{III}(\varrho_1, \theta_1, \theta_2, \theta_3) = U_{III}(\varrho_1, -\theta_1, -\theta_2, \theta_3) \text{diag}(1, 1, -1). \quad (5.30)$$

Hence three mixing patterns are obtained after all the six row permutations are considered,

$$U_{III,1} = U_{III}, \quad U_{III,2} = P_{132}U_{III}, \quad U_{III,3} = P_{213}U_{III}. \quad (5.31)$$

For the mixing matrix $U_{III,1}$, we can extract the mixing parameters in the usual way and find

$$\begin{aligned} \sin^2 \theta_{13} &= \frac{1}{2} (\sin^2 \theta_2 + \cos^2 \theta_1 \cos^2 \theta_2 - \cos \theta_1 \sin 2\theta_2 \cos \varrho_1), \\ \sin^2 \theta_{12} &= \sin^2 \theta_3 + \frac{\sin 2\theta_3 (2 \sin \theta_1 \cos \theta_2 \cos \varrho_1 + \sin 2\theta_1 \sin \theta_2) + 2 \sin^2 \theta_1 \cos 2\theta_3}{2 - \sin^2 \theta_2 - \cos^2 \theta_1 \cos^2 \theta_2 + \cos \theta_1 \sin 2\theta_2 \cos \varrho_1}, \\ \sin^2 \theta_{23} &= \frac{2 \sin^2 \theta_1 \cos^2 \theta_2}{2 - \sin^2 \theta_2 - \cos^2 \theta_1 \cos^2 \theta_2 + \cos \theta_1 \sin 2\theta_2 \cos \varrho_1}, \\ J_{CP} &= \frac{1}{16} \sin \theta_1 \cos \theta_2 \sin \varrho_1 [4 \sin 2\theta_1 \sin \theta_2 \cos 2\theta_3 + (1 + 3 \cos 2\theta_1 + 2 \sin^2 \theta_1 \cos 2\theta_2) \sin 2\theta_3], \end{aligned} \quad (5.32)$$

which have the symmetry transformation $(\theta_1, \theta_2, \theta_3) \rightarrow (\pi - \theta_1, \pi - \theta_2, \pi - \theta_3)$. The parameter value of $\varrho_1 = 0$ is always admissible, and the resulting lepton mixing matrix is the same as U_I if the possible shifts in $\theta_{1,2,3}$ are taken into account. Consequently the lepton mixing angles in the experimentally preferred range can be achieved for appropriate choices of the parameters $\theta_{1,2,3}$. However, both Dirac phase δ and Majorana phase ϕ would be determined to be trivial. The smallest value of the index n which is capable of accommodating the experimental data and nontrivial CP violating phases is $n = 6$ with $\varrho_1 = \pi/6$ up to the symmetry transformations shown in Eq. (5.29). Please see table 6 for the corresponding results of the χ^2 analysis. We see that the atmospheric mixing angle deviates from maximal mixing with $\sin^2 \theta_{23} = 0.577$ (0.641) for NO (IO) at the best fit point where the χ^2 function reaches a global minimum, and the Dirac CP phase is approximately maximal with $|\sin \delta| = 0.985$ (0.983). This result is consistent with the weak evidence of maximal Dirac CP violation reported by T2K [10] and NO ν A [11] and global data fitting [4–8], and it can be tested in forthcoming neutrino oscillation experiments [58, 59]. As regards the leptogenesis, the relevant CP invariants are of the form

$$\begin{aligned} I_{\text{NO}}^e &= -I_{\text{NO}}^\tau = -\frac{1}{2} (\cos \theta_1 \sin \theta_3 + \sin \theta_1 \sin \theta_2 \cos \theta_3) \sin \varrho_1, \quad I_{\text{NO}}^\mu = 0, \\ I_{\text{IO}}^e &= -I_{\text{IO}}^\tau = \frac{1}{2} \sin \theta_1 \cos \theta_2 \sin \varrho_1, \quad I_{\text{IO}}^\mu = 0, \end{aligned} \quad (5.33)$$

One sees that the all the lepton asymmetry ϵ_α would be vanishing for $\varrho_1 = 0$ such that the cosmic baryon asymmetry can not be generated. In the case of $\varrho_1 = \pi/6$, the numerical results of Y_B versus ϑ are shown in figure 5. We notice that the correct value of the baryon asymmetry can be obtained for particular values of ϑ except in the case of R-3rd with NO spectrum and R-1st of IO.

For the second mixing pattern $U_{III,2}$, the three lepton mixing angles and Jarlskog invariant are determined to be

$$\begin{aligned} \sin^2 \theta_{13} &= \frac{1}{2} (\sin^2 \theta_2 + \cos^2 \theta_1 \cos^2 \theta_2 - \cos \theta_1 \sin 2\theta_2 \cos \varrho_1), \\ \sin^2 \theta_{12} &= \sin^2 \theta_3 + \frac{\sin 2\theta_3 (2 \sin \theta_1 \cos \theta_2 \cos \varrho_1 + \sin 2\theta_1 \sin \theta_2) + 2 \sin^2 \theta_1 \cos 2\theta_3}{2 - \sin^2 \theta_2 - \cos^2 \theta_1 \cos^2 \theta_2 + \cos \theta_1 \sin 2\theta_2 \cos \varrho_1}, \\ \sin^2 \theta_{23} &= \frac{1 - \sin^2 \theta_1 \cos^2 \theta_2 + \cos \theta_1 \sin 2\theta_2 \cos \varrho_1}{2 - \sin^2 \theta_2 - \cos^2 \theta_1 \cos^2 \theta_2 + \cos \theta_1 \sin 2\theta_2 \cos \varrho_1}, \\ J_{CP} &= -\frac{1}{16} \sin \theta_1 \cos \theta_2 \sin \varrho_1 [4 \sin 2\theta_1 \sin \theta_2 \cos 2\theta_3 + (1 + 3 \cos 2\theta_1 + 2 \sin^2 \theta_1 \cos 2\theta_2) \sin 2\theta_3]. \end{aligned} \quad (5.34)$$

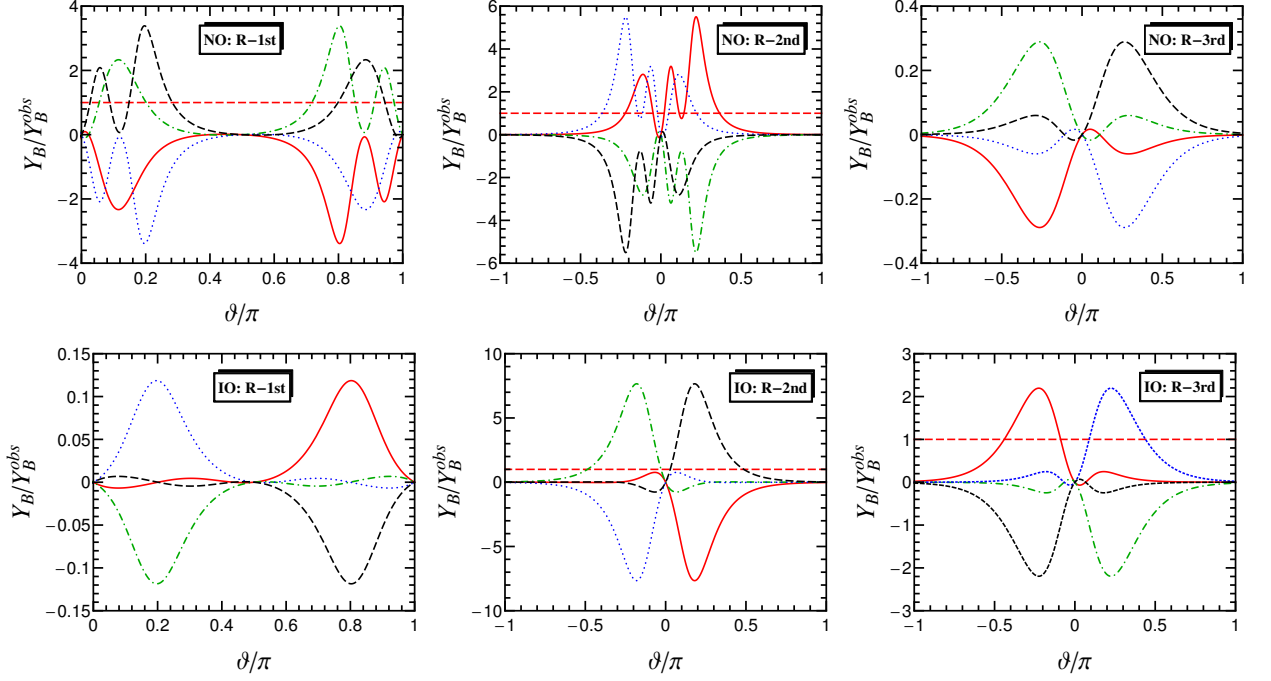


Figure 5: Y_B/Y_B^{obs} as a function of the parameter ϑ for the mixing pattern $U_{III,1}$ with $\varrho_1 = \pi/6$, where we choose the RH neutrino mass $M_1 = 5 \times 10^{11}$ GeV. The red solid, green dash-dotted, blue dotted and black dashed lines correspond to the four best fitting points shown in table 6 one by one. The horizontal red dashed line represents the experimental measured value Y_B^{obs} . The neutrino mass spectrum is NO and IO in the first row and the second row respectively. The panels in the left, middle and right columns are for the three admissible forms of the R -matrix such as R-1st, R-2nd and R-3rd respectively.

As shown in table 6, agreement with the experimental data can be achieved for both $\varrho_1 = 0$ and $\varrho_1 = \pi/6$. Regarding the CP invariants in leptogenesis, we get

$$\begin{aligned} I_{NO}^e &= -I_{NO}^\mu = -\frac{1}{2}(\cos \theta_1 \sin \theta_3 + \sin \theta_1 \sin \theta_2 \cos \theta_3) \sin \varrho_1, & I_{NO}^\tau &= 0, \\ I_{IO}^e &= -I_{IO}^\mu = \frac{1}{2} \sin \theta_1 \cos \theta_2 \sin \varrho_1, & I_{IO}^\tau &= 0, \end{aligned} \quad (5.35)$$

which implies $I_{NO}^e + I_{NO}^\mu = 0$ and $I_{IO}^e + I_{IO}^\mu = 0$. Hence the summation of the CP asymmetry in the electron and muon flavors would vanish, i.e. $\epsilon_2 \equiv \epsilon_e + \epsilon_\mu = 0$. As a consequence, Y_B would be predicted to be zero in the mass window $10^9 \text{ GeV} \leq M_1 \leq 10^{12} \text{ GeV}$ unless the postulated residual symmetry is broken by non-leading order corrections arising from higher dimensional operators. For the third possible PMNS mixing matrix $U_{III,3}$, the lepton mixing parameters read as

$$\begin{aligned} \sin^2 \theta_{13} &= \sin^2 \theta_1 \cos^2 \theta_2, \\ \sin^2 \theta_{12} &= \frac{(\cos \theta_1 \cos \theta_3 - \sin \theta_1 \sin \theta_2 \sin \theta_3)^2}{1 - \sin^2 \theta_1 \cos^2 \theta_2}, \\ \sin^2 \theta_{23} &= \frac{1}{2} - \frac{\cos \theta_1 \sin 2\theta_2 \cos \varrho_1}{2 - 2 \sin^2 \theta_1 \cos^2 \theta_2}, \\ J_{CP} &= -\frac{1}{16} \sin \theta_1 \cos \theta_2 \sin \varrho_1 \left[4 \sin 2\theta_1 \sin \theta_2 \cos 2\theta_3 + (1 + 3 \cos 2\theta_1 + 2 \sin^2 \theta_1 \cos 2\theta_2) \sin 2\theta_3 \right]. \end{aligned} \quad (5.36)$$

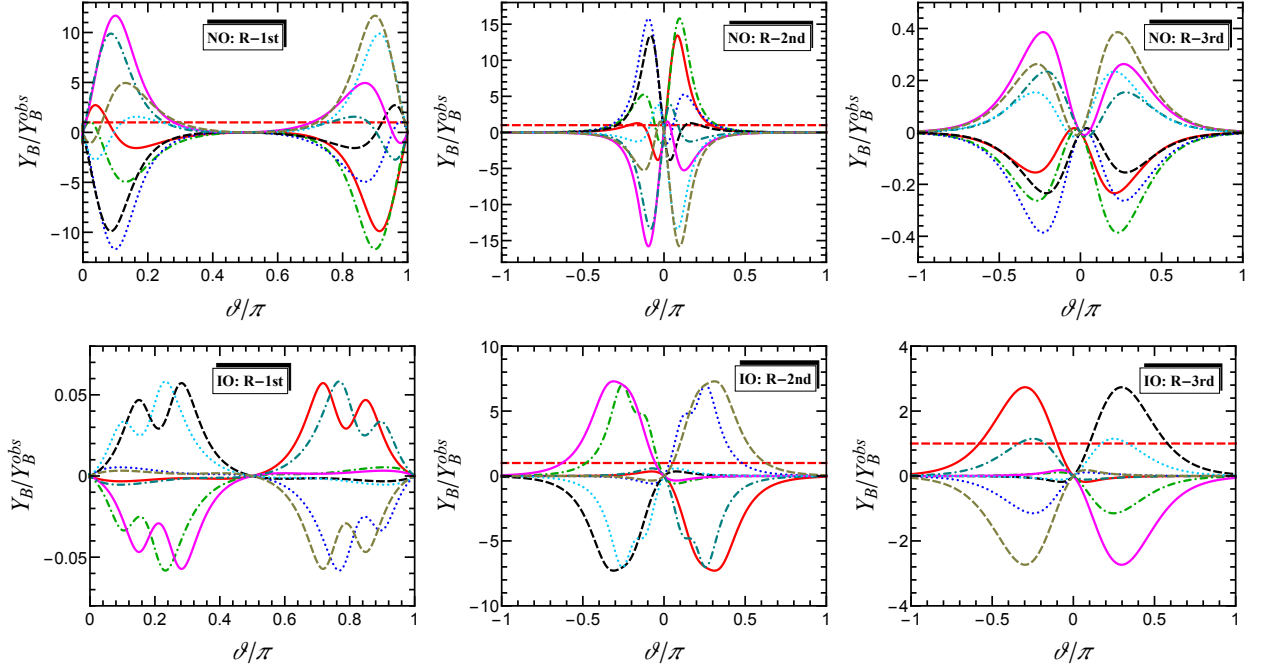


Figure 6: Y_B/Y_B^{obs} as a function of the parameter ϑ for the mixing pattern $U_{III,3}$ with $\varrho_1 = \pi/3$, where we choose the RH neutrino mass $M_1 = 5 \times 10^{11}$ GeV. The red solid, green dash-dotted, blue dotted, black dashed, pink solid, cyan dash-dotted, dark green dotted and brown dashed lines correspond to the eight best fitting points shown in table 6 one by one. The horizontal red dashed line represents the experimental measured value Y_B^{obs} . The neutrino mass spectrum is NO and IO in the first row and the second row respectively. The panels in the left, middle and right columns are for the three admissible forms of the R -matrix such as R-1st, R-2nd and R-3rd respectively.

The rephase invariants I_{NO}^α and I_{IO}^α are of the form

$$\begin{aligned} I_{NO}^\mu &= -I_{NO}^\tau = -\frac{1}{2}(\cos \theta_1 \sin \theta_3 + \sin \theta_1 \sin \theta_2 \cos \theta_3) \sin \varrho_1, & I_{NO}^e &= 0, \\ I_{IO}^\mu &= -I_{IO}^\tau = \frac{1}{2} \sin \theta_1 \cos \theta_2 \sin \varrho_1, & I_{IO}^e &= 0. \end{aligned} \quad (5.37)$$

In the case of $n = 2$, the parameter ϱ_1 can be either 0 or $\pi/2$. We find that experimental data on lepton mixing angles can be accommodated well for both $\varrho_1 = 0$ and $\varrho_1 = \pi/2$. The mixing pattern $U_{III,3}$ with $\varrho_1 = 0$ is equivalent to U_I in Eq. (5.16), the Dirac as well as Majorana CP phases are trivial, and consequently the baryon asymmetry can not be generated. The mixing matrix $U_{III,3}$ for $\varrho_1 = \pi/2$ is related to $U_{II,3}$ as follow,

$$U_{III,3}(\varrho_1 = \pi/2, \theta_1, \theta_2, \theta_3) = U_{II,3}(\theta'_1, \theta'_2, \theta'_3), \quad (5.38)$$

where $\theta'_{1,2,3}$ are defined through $O_{3 \times 3}(\theta'_1, \theta'_2, \theta'_3) = P_{231} O_{3 \times 3}(\theta_1, \theta_2, \theta_3)$. Hence $U_{III,3}$ with $\varrho_1 = \pi/2$ and $U_{II,3}$ lead to the same predictions for lepton mixing parameters and Y_B . Furthermore, new mixing pattern can be obtained from the $\Delta(6 \cdot 3^2) = \Delta(54)$ group for $\varrho_1 = \pi/3$. Note that $\varrho_1 = 2\pi/3$ leads to the same mixing matrix as $\varrho_1 = \pi/3$ after the shift of $\theta_{1,2,3}$ is considered. As shown in table 6, the best fit values [4] of the three mixing angles can be achieved for certain values of the parameters $\theta_{1,2,3}$. The corresponding predictions for Y_B as a function of ϑ are plotted in figure 6. The observed matter-antimatter asymmetry could be reproduced except for the cases of NO:R-3rd and IO:R-1st.

(IV) $G_l = \langle bc^s d^t \rangle$, $X_\nu = \rho_3(bc^x d^{-x})$

In the same manner as previous cases, we find the lepton mixing matrix is given by

$$U_{IV} = P_l \begin{pmatrix} 0 & \cos \varrho_2 & \sin \varrho_2 \\ 1 & 0 & 0 \\ 0 & -\sin \varrho_2 & \cos \varrho_2 \end{pmatrix} O_{3 \times 3}(\theta_1, \theta_2, \theta_3) \hat{X}_\nu^{-\frac{1}{2}} \\ = P_l P_{213} O_{3 \times 3}(\theta_1 + \varrho_2, \theta_2, \theta_3) \hat{X}_\nu^{-\frac{1}{2}}, \quad (5.39)$$

where P_l is a generic 3×3 permutation matrix, and the contributions of P_l and P_{213} can be absorbed into the real orthogonal matrix $O_{3 \times 3}$. The parameter ϱ_2 is fixed by the chosen residual symmetry as

$$\varrho_2 = -\frac{s+t}{2n}\pi \quad (5.40)$$

whose possible values are

$$\varrho_2 \pmod{2\pi} = 0, \frac{1}{2n}\pi, \frac{2}{2n}\pi, \dots, \frac{4n-1}{2n}\pi. \quad (5.41)$$

After the relabeling of $P_l P_{213} \rightarrow P_l$ and $\theta_1 + \varrho_1 \rightarrow \theta_1$ is taken into account, the mixing matrix U_{IV} would coincide with U_I shown in Eq. (5.16). As a result, the predictions for mixing parameters and leptogenesis are exactly the same as case I. The experimentally preferred values of the lepton mixing angles can be accommodated, the Dirac CP phase δ is trivial, and the cosmic baryon asymmetry Y_B is predicted to be vanishing without higher order corrections.

(V) $G_l = \langle ac^s d^t \rangle$, $X_\nu = \rho_3(c^x d^y)$

Combining the unitary transformations U_l for $G_l = \langle ac^s d^t \rangle$ shown in table 5 and U_ν in Eq. (5.13), we find that the PMNS mixing matrix is of the form

$$U_V = \frac{1}{\sqrt{3}} \begin{pmatrix} e^{i\varrho_3} & 1 & e^{i\varrho_4} \\ \omega^2 e^{i\varrho_3} & 1 & \omega e^{i\varrho_4} \\ \omega e^{i\varrho_3} & 1 & \omega^2 e^{i\varrho_4} \end{pmatrix} O_{3 \times 3}(\theta_1, \theta_2, \theta_3) \hat{X}_\nu^{-\frac{1}{2}}, \quad (5.42)$$

up to permutations of rows, where ϱ_3 and ϱ_4 are determined by residual symmetry,

$$\varrho_3 = \frac{2s-2t+2x-y}{n}\pi, \quad \varrho_4 = \frac{-2t+x-2y}{n}\pi, \quad (5.43)$$

which can independently take the values

$$\varrho_3, \varrho_4 \pmod{2\pi} = 0, \frac{1}{n}\pi, \frac{2}{n}\pi, \dots, \frac{2n-1}{n}\pi. \quad (5.44)$$

We observe that the mixing matrix U_V has the following properties

$$U_V(\varrho_3 + \pi, \varrho_4, \theta_1, \theta_2, \theta_3) = U_V(\varrho_3, \varrho_4, \theta_1, -\theta_2, -\theta_3) \text{diag}(-1, 1, 1), \\ U_V(\varrho_3, \varrho_4 + \pi, \theta_1, \theta_2, \theta_3) = U_V(\varrho_3, \varrho_4, -\theta_1, -\theta_2, \theta_3) \text{diag}(1, 1, -1). \quad (5.45)$$

Consequently the fundamental regions of the parameters ϱ_3 and ϱ_4 can be taken to be $[0, \pi)$. Exchanging the second and the third rows of U_V leads to the same mixing pattern as swapping ϱ_3 and ϱ_4 , i.e.

$$P_{132} U_V(\varrho_3, \varrho_4, \theta_1, \theta_2, \theta_3) = U_V(\varrho_4, \varrho_3, \theta'_1, \theta'_2, \theta'_3), \quad (5.46)$$

where $\theta'_{1,2,3}$ fulfill $O_{3 \times 3}(\theta'_1, \theta'_2, \theta'_3) = P_{321} O_{3 \times 3}(\theta_1, \theta_2, \theta_3)$. Hence it is enough to only consider three out of the six possible row permutations,

$$U_{V,1} = U_V, \quad U_{V,2} = P_{213} U_V, \quad U_{V,3} = P_{231} U_V. \quad (5.47)$$

For the case of $U_{V,1}$, we can obtain the following expressions for the mixing angles and the Jarlskog invariant,

$$\begin{aligned}
\sin^2 \theta_{13} &= \frac{1}{3} \left[\sin 2\theta_2 (\cos \theta_1 \cos(\varrho_3 - \varrho_4) + \sin \theta_1 \cos \varrho_3) + \sin 2\theta_1 \cos^2 \theta_2 \cos \varrho_4 + 1 \right], \\
\sin^2 \theta_{12} &= \sin^2 \theta_3 + \frac{\sin 2\theta_3 (\cos \theta_1 \cos \theta_2 \cos \varrho_3 - \sin \theta_1 \cos \theta_2 \cos(\varrho_3 - \varrho_4) - \cos 2\theta_1 \sin \theta_2 \cos \varrho_4)}{2 - \sin 2\theta_2 (\cos \theta_1 \cos(\varrho_3 - \varrho_4) + \sin \theta_1 \cos \varrho_3) - \sin 2\theta_1 \cos^2 \theta_2 \cos \varrho_4} \\
&\quad + \frac{\cos 2\theta_3 (1 - \sin 2\theta_1 \cos \varrho_4)}{2 - \sin 2\theta_2 (\cos \theta_1 \cos(\varrho_3 - \varrho_4) + \sin \theta_1 \cos \varrho_3) - \sin 2\theta_1 \cos^2 \theta_2 \cos \varrho_4}, \\
\sin^2 \theta_{23} &= \frac{\sin 2\theta_2 (\cos \theta_1 \sin(\varrho_3 - \varrho_4 + \frac{\pi}{6}) + \sin \theta_1 \cos(\varrho_3 + \frac{\pi}{3})) + \cos^2 \theta_2 \sin 2\theta_1 \cos(\varrho_4 - \frac{\pi}{3}) - 1}{\sin 2\theta_2 (\cos \theta_1 \cos(\varrho_3 - \varrho_4) + \sin \theta_1 \cos \varrho_3) + \sin 2\theta_1 \cos^2 \theta_2 \cos \varrho_4 - 2}, \\
J_{CP} &= \frac{1}{6\sqrt{3}} \left[\cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 + \sin 2\theta_1 \sin \theta_2 (1 - 3/2 \cos^2 \theta_2) \sin 2\theta_3 \right. \\
&\quad \left. + \cos \theta_2 (\sin 2\theta_3 (\cos^2 \theta_1 - \sin^2 \theta_1 \sin^2 \theta_2) + \sin 2\theta_1 \sin \theta_2 \cos 2\theta_3) [\sin \theta_1 \cos(\varrho_3 + \varrho_4) \right. \\
&\quad \left. - \cos \theta_1 \cos(\varrho_3 - 2\varrho_4)] + \cos^3 \theta_2 \sin 2\theta_3 (\cos \theta_1 \cos(\varrho_3 - 2\varrho_4) - \tan \theta_2 \cos(2\varrho_3 - \varrho_4)) \right]. \quad (5.48)
\end{aligned}$$

Moreover, the CP invariants I_{NO}^α and I_{IO}^α are given by

$$\begin{aligned}
I_{\text{NO}}^e &= \frac{1}{3} [\sin(\varrho_4 - \varrho_3) (\sin \theta_1 \sin \theta_2 \cos \theta_3 + \cos \theta_1 \sin \theta_3) + \cos \theta_2 \cos \theta_3 \sin \varrho_4 \\
&\quad + \sin \varrho_3 (\cos \theta_1 \sin \theta_2 \cos \theta_3 - \sin \theta_1 \sin \theta_3)], \\
I_{\text{NO}}^\mu &= \frac{1}{3} \left[\sin(\varrho_3 - \varrho_4 - \frac{\pi}{3}) (\sin \theta_1 \sin \theta_2 \cos \theta_3 + \cos \theta_1 \sin \theta_3) + \cos \theta_2 \cos \theta_3 \sin(\frac{\pi}{3} - \varrho_4) \right. \\
&\quad \left. + \sin(\varrho_3 + \frac{\pi}{3}) (\sin \theta_1 \sin \theta_3 - \cos \theta_1 \sin \theta_2 \cos \theta_3) \right], \\
I_{\text{IO}}^e &= \frac{1}{3} (\sin \theta_1 \cos \theta_2 \sin(\varrho_3 - \varrho_4) - \cos \theta_1 \cos \theta_2 \sin \varrho_3 + \sin \theta_2 \sin \varrho_4), \\
I_{\text{IO}}^\mu &= \frac{1}{3} \left(\sin \theta_1 \cos \theta_2 \sin(\frac{\pi}{3} - \varrho_3 + \varrho_4) + \cos \theta_1 \cos \theta_2 \sin(\varrho_3 + \frac{\pi}{3}) + \sin \theta_2 \sin(\frac{\pi}{3} - \varrho_4) \right), \\
I_{\text{NO}}^\tau &= -(I_{\text{NO}}^e + I_{\text{NO}}^\mu), \quad I_{\text{IO}}^\tau = -(I_{\text{IO}}^e + I_{\text{IO}}^\mu). \quad (5.49)
\end{aligned}$$

For the $\Delta(6n^2)$ group with the smallest index $n = 2$, the values of ϱ_3 and ϱ_4 can be 0 and $\pi/2$. Utilizing the equivalence condition of Eq. (5.10), we find two independent mixing patterns with $\varrho_3 = \varrho_4 = 0$ and $\varrho_3 = 0, \varrho_4 = \pi/2$. Moreover, the mixing matrix $U_{V,1}$ for $\varrho_3 = \varrho_4 = 0$ is the same as $U_{II,3}$ if the possible shifts of $\theta_{1,2,3}$ is considered. In the case of $\varrho_3 = 0$ and $\varrho_4 = \pi/2$, the results of the χ^2 analysis are summarized in table 6, and the predictions for Y_B are plotted in figure 7. For the flavor group $\Delta(6 \cdot 3^2) = \Delta(54)$, the possible values of ϱ_3 and ϱ_4 are 0, $\pi/3$ and $2\pi/3$. We can obtain three phenomenologically viable mixing patterns corresponding to $(\varrho_3, \varrho_4) = (0, 0)$, $(0, \pi/3)$, $(0, 2\pi/3)$. Note that $U_{V,1}$ for $(\varrho_3, \varrho_4) = (0, 2\pi/3)$ is equivalent to the complex conjugate of $U_{V,1}$ for $(\varrho_3, \varrho_4) = (0, \pi/3)$. The best fit values of the three lepton mixing angles can be reproduced for particular values of $\theta_{1,2,3}$ in the case of $(\varrho_3, \varrho_4) = (0, \pi/3)$, the resulting predictions for CP violation phases are listed in table 6, and the variation of Y_B with respect to ϑ is plotted in figure 8.

Then we proceed to discuss the second permutation $U_{V,2}$, we can straightforwardly extract the mixing parameters and find

$$\begin{aligned}
\sin^2 \theta_{13} &= \frac{1}{3} \left[1 - \sin 2\theta_2 (\cos \theta_1 \cos(\varrho_3 - \varrho_4 - \frac{\pi}{3}) + \sin \theta_1 \cos(\varrho_3 + \frac{\pi}{3})) - \sin 2\theta_1 \cos^2 \theta_2 \cos(\varrho_4 - \frac{\pi}{3}) \right], \\
\sin^2 \theta_{12} &= \sin^2 \theta_3 + \frac{\cos 2\theta_3 (\sin 2\theta_1 \cos(\varrho_4 - \frac{\pi}{3}) + 1)}{2 + \sin 2\theta_2 (\cos \theta_1 \cos(\varrho_3 - \varrho_4 - \frac{\pi}{3}) + \sin \theta_1 \cos(\varrho_3 + \frac{\pi}{3})) + \sin 2\theta_1 \cos^2 \theta_2 \cos(\varrho_4 - \frac{\pi}{3})}, \\
&\quad + \frac{\sin 2\theta_3 (\sin \theta_1 \cos \theta_2 \cos(\varrho_3 - \varrho_4 - \frac{\pi}{3}) - \cos \theta_1 \cos \theta_2 \cos(\varrho_3 + \frac{\pi}{3}) + \cos 2\theta_1 \sin \theta_2 \cos(\varrho_4 - \frac{\pi}{3}))}{2 + \sin 2\theta_2 (\cos \theta_1 \cos(\varrho_3 - \varrho_4 - \frac{\pi}{3}) + \sin \theta_1 \cos(\varrho_3 + \frac{\pi}{3})) + \sin 2\theta_1 \cos^2 \theta_2 \cos(\varrho_4 - \frac{\pi}{3})},
\end{aligned}$$

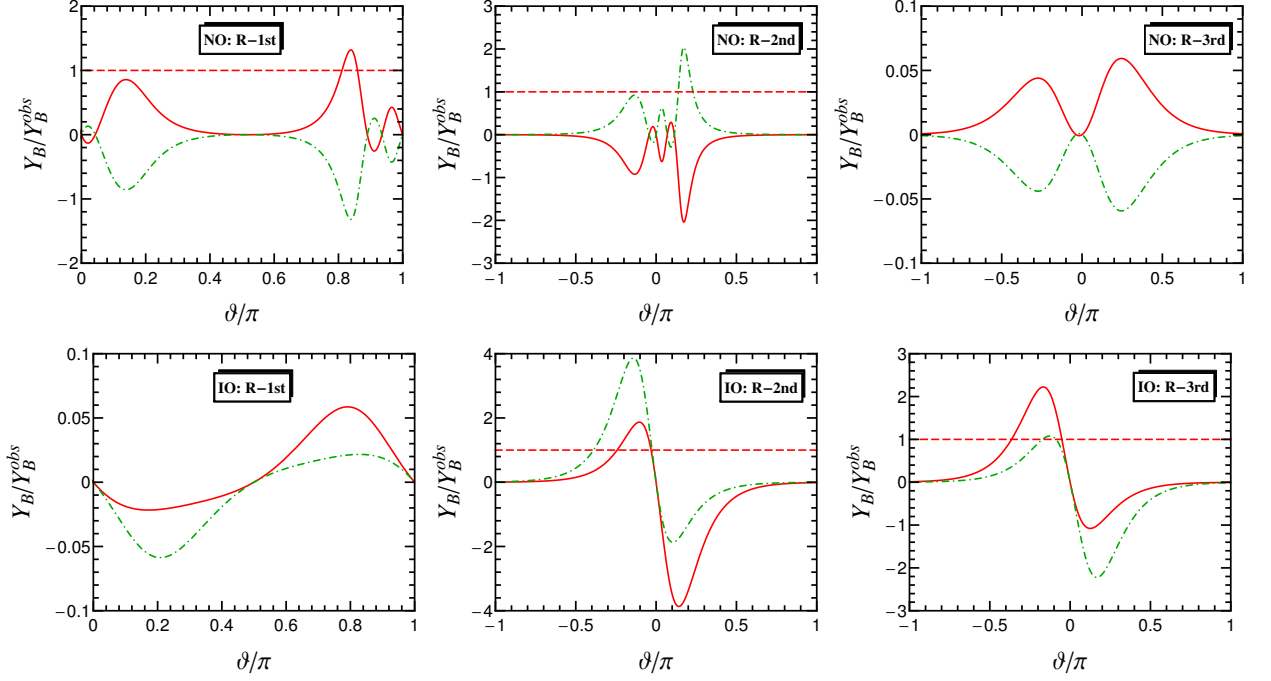


Figure 7: Y_B/Y_B^{obs} as a function of the parameter ϑ for the mixing pattern $U_{V,1}$ with $\varrho_3 = 0$ and $\varrho_4 = \pi/2$, where we choose the RH neutrino mass $M_1 = 5 \times 10^{11}$ GeV. The red solid and green dash-dotted lines correspond to the two best fitting points shown in table 6 one by one. The horizontal red dashed line represents the experimental measured value Y_B^{obs} . The neutrino mass spectrum is NO and IO in the first row and the second row respectively. The panels in the left, middle and right columns are for the three admissible forms of the R -matrix such as R-1st, R-2nd and R-3rd respectively.

$$\sin^2 \theta_{23} = \frac{\sin 2\theta_2 (\cos \theta_1 \cos(\varrho_3 - \varrho_4) + \sin \theta_1 \cos \varrho_3) + \sin 2\theta_1 \cos^2 \theta_2 \cos \varrho_4 + 1}{2 + \sin 2\theta_2 (\cos \theta_1 \cos(\varrho_3 - \varrho_4 - \frac{\pi}{3}) + \sin \theta_1 \cos(\varrho_3 + \frac{\pi}{3})) + \sin 2\theta_1 \cos^2 \theta_2 \cos(\varrho_4 - \frac{\pi}{3})},$$

$$J_{CP} = -\frac{1}{6\sqrt{3}} \left[\cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 + \sin 2\theta_1 \sin \theta_2 (1 - 3/2 \cos^2 \theta_2) \sin 2\theta_3 \right. \\ \left. + \cos \theta_2 (\sin 2\theta_3 (\cos^2 \theta_1 - \sin^2 \theta_1 \sin^2 \theta_2) + \sin 2\theta_1 \sin \theta_2 \cos 2\theta_3) [\sin \theta_1 \cos(\varrho_3 + \varrho_4) \right. \\ \left. - \cos \theta_1 \cos(\varrho_3 - 2\varrho_4)] + \cos^3 \theta_2 \sin 2\theta_3 (\cos \theta_1 \cos(\varrho_3 - 2\varrho_4) - \tan \theta_2 \cos(2\varrho_3 - \varrho_4)) \right]. \quad (5.50)$$

Since $U_{V,2}$ and $U_{V,1}$ are related through the permutation of the first and second rows, the rephasing invariants $I_{NO,IO}^\alpha$ can be obtained from Eq. (5.49) by interchanging the expressions of $I_{NO,IO}^e$ and $I_{NO,IO}^\mu$. The third mixing matrix $U_{V,3}$ can be easily obtained by exchanging the second and third rows of $U_{V,2}$. As a consequence, $U_{V,2}$ and $U_{V,3}$ lead to the same reactor and solar mixing angles, while the atmospheric one changes from θ_{23} to $\pi/2 - \theta_{23}$, i.e. $\sin^2 \theta_{23}$ is replaced by $\cos^2 \theta_{23}$ in Eq. (5.50), and the Dirac phase changes from δ to $\pi + \delta$ such that the overall sign of the Jarlskog invariant J_{CP} becomes opposite. Furthermore, the CP invariants can be obtained from Eq. (5.49) by replacing $I_{NO,IO}^e \rightarrow I_{NO,IO}^\tau$, $I_{NO,IO}^\mu \rightarrow I_{NO,IO}^e$ and $I_{NO,IO}^\tau \rightarrow I_{NO,IO}^\mu$. Note that both $U_{V,2}$ and $U_{V,3}$ can not accommodate the experimental data on the mixing angles for $n = 2$, and they give rise to the same mixing patterns as $U_{V,1}$ in the case of $n = 3$.

(VI) $G_l = \langle ac^s d^t \rangle$, $X_\nu = \rho_3(bc^x d^{-x})$

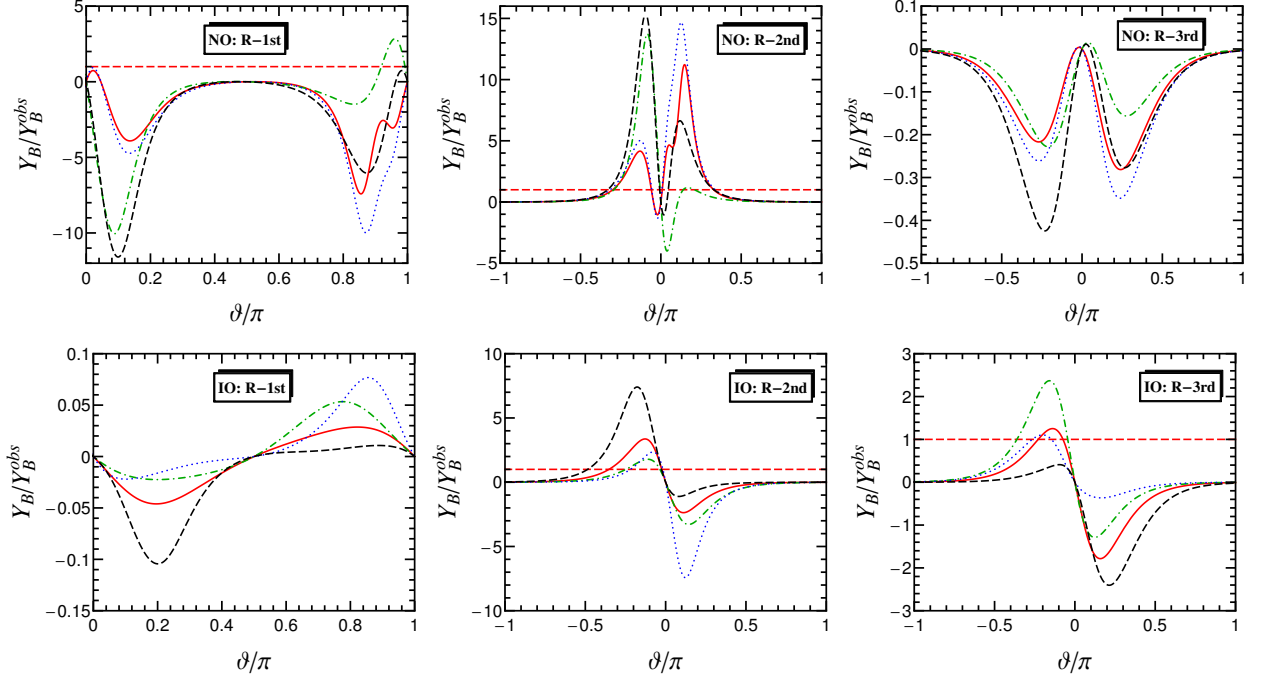


Figure 8: Y_B/Y_B^{obs} as a function of the parameter ϑ for the mixing pattern $U_{V,1}$ with $\varrho_3 = 0$ and $\varrho_4 = \pi/3$, where we choose the RH neutrino mass $M_1 = 5 \times 10^{11}$ GeV. The red solid, green dash-dotted, blue dotted and black dashed lines correspond to the four best fitting points shown in table 6 one by one. The horizontal red dashed line represents the experimental measured value Y_B^{obs} . The neutrino mass spectrum is NO and IO in the first row and the second row respectively. The panels in the left, middle and right columns are for the three admissible forms of the R -matrix such as R-1st, R-2nd and R-3rd respectively.

In this case, the PMNS mixing matrix takes the following form

$$\begin{aligned}
 U_{VI} &= \sqrt{\frac{2}{3}} \begin{pmatrix} \frac{e^{i\varrho_5}}{\sqrt{2}} & \sin \varrho_6 & \cos \varrho_6 \\ -\frac{e^{i\varrho_5}}{\sqrt{2}} & \cos(\frac{\pi}{6} - \varrho_6) & \sin(\frac{\pi}{6} - \varrho_6) \\ \frac{e^{i\varrho_5}}{\sqrt{2}} & \cos(\varrho_6 + \frac{\pi}{6}) & -\sin(\varrho_6 + \frac{\pi}{6}) \end{pmatrix} O_{3 \times 3}(\theta_1, \theta_2, \theta_3) \hat{X}_\nu^{-\frac{1}{2}} \\
 &= \sqrt{\frac{2}{3}} \begin{pmatrix} \frac{e^{i\varrho_5}}{\sqrt{2}} & 0 & 1 \\ -\frac{e^{i\varrho_5}}{\sqrt{2}} & \cos \frac{\pi}{6} & \sin \frac{\pi}{6} \\ \frac{e^{i\varrho_5}}{\sqrt{2}} & \cos \frac{\pi}{6} & -\sin \frac{\pi}{6} \end{pmatrix} O_{3 \times 3}(\theta_1 - \varrho_6, \theta_2, \theta_3) \hat{X}_\nu^{-\frac{1}{2}}, \quad (5.51)
 \end{aligned}$$

where the discrete parameters ϱ_5 and ϱ_6 depend on the choice of the residual symmetry as

$$\varrho_5 = \frac{-s + 2t - 3x}{n} \pi, \quad \varrho_6 = \frac{s}{n} \pi \quad (5.52)$$

whose values can be

$$\varrho_5, \varrho_6 \pmod{2\pi} = 0, \frac{1}{n}\pi, \frac{2}{n}\pi, \dots, \frac{2n-1}{n}\pi. \quad (5.53)$$

From Eq. (5.51) we can see that the parameter ϱ_6 is irrelevant since it can be absorbed into the free parameter θ_1 . Furthermore we find that U_{VI} has several symmetry properties,

$$\begin{aligned}
 P_{132} U_{VI}(\varrho_5, \varrho_6, \theta_1, \theta_2, \theta_3) &= \text{diag}(1, -1, -1) U_{VI}(\varrho_5, -\varrho_6, -\theta_1, \theta_2, -\theta_3) \text{diag}(1, -1, 1), \\
 P_{312} U_{VI}(\varrho_5, \varrho_6, \theta_1, \theta_2, \theta_3) &= \text{diag}(1, -1, -1) U_{VI}(\varrho_5, \varrho_6 + \frac{2\pi}{3}, \theta_1, \theta_2, \theta_3), \\
 P_{231} U_{VI}(\varrho_5, \varrho_6, \theta_1, \theta_2, \theta_3) &= \text{diag}(-1, -1, 1) U_{VI}(\varrho_5, \varrho_6 - \frac{2\pi}{3}, \theta_1, \theta_2, \theta_3), \quad (5.54)
 \end{aligned}$$

and

$$U_{VI}(\varrho_5 + \pi, \varrho_6, \theta_1, \theta_2, \theta_3) = U_{VI}(\varrho_5, \varrho_6, \theta_1, -\theta_2, -\theta_3) \text{diag}(-1, 1, 1). \quad (5.55)$$

Eq. (5.54) implies that the six possible row permutations lead to the same mixing pattern if all possible values of ϱ_5 and ϱ_6 are considered, and Eq. (5.55) indicates that the fundamental region of ϱ_5 is $[0, \pi)$. We can read off the mixing parameters from the mixing matrix U_{VI} in Eq. (5.51) as follows

$$\begin{aligned} \sin^2 \theta_{13} &= \frac{1}{3} \left(1 + \cos 2\theta_1 \cos^2 \theta_2 + \sqrt{2} \cos \theta_1 \sin 2\theta_2 \cos \varrho_5 \right), \\ \sin^2 \theta_{12} &= \sin^2 \theta_3 + \frac{\sin \theta_1 (2 \sin \theta_1 \cos 2\theta_3 - \sin 2\theta_3 (\sqrt{2} \cos \theta_2 \cos \varrho_5 - 2 \cos \theta_1 \sin \theta_2))}{2 - \cos 2\theta_1 \cos^2 \theta_2 - \sqrt{2} \cos \theta_1 \sin 2\theta_2 \cos \varrho_5}, \\ \sin^2 \theta_{23} &= \frac{1 - \cos (2\theta_1 + \pi/3) \cos^2 \theta_2 - \sqrt{2} \sin(\theta_1 + \pi/6) \sin 2\theta_2 \cos \varrho_5}{2 - \cos 2\theta_1 \cos^2 \theta_2 - \sqrt{2} \cos \theta_1 \sin 2\theta_2 \cos \varrho_5}, \\ J_{CP} &= \frac{\cos \theta_2 \sin 2\theta_3 \sin \varrho_5 [4 \sin 3\theta_1 \sin \theta_2 \cot 2\theta_3 - \cos 3\theta_1 (\cos 2\theta_2 - 3) - 2\sqrt{2} \sin 2\theta_2 \cos \varrho_5]}{12\sqrt{6}}, \end{aligned} \quad (5.56)$$

where the redefinition of $\theta_1 \rightarrow \theta_1 + \varrho_6$ is used. Moreover the rephasing invariants involved in leptogenesis are found to be of the form

$$\begin{aligned} I_{\text{NO}}^e &= -\frac{\sqrt{2}}{3} \sin \varrho_5 (\cos \theta_1 \sin \theta_3 + \sin \theta_1 \sin \theta_2 \cos \theta_3), \\ I_{\text{NO}}^\mu &= \frac{\sqrt{2}}{3} \sin \varrho_5 [\sin(\theta_1 + \pi/6) \sin \theta_3 + \sin(\theta_1 - \pi/3) \sin \theta_2 \cos \theta_3] \\ I_{\text{IO}}^e &= \frac{\sqrt{2}}{3} \sin \theta_1 \cos \theta_2 \sin \varrho_5, \\ I_{\text{IO}}^\mu &= -\frac{\sqrt{2}}{3} \sin(\theta_1 - \pi/3) \cos \theta_2 \sin \varrho_5, \\ I_{\text{NO}}^\tau &= -(I_{\text{NO}}^e + I_{\text{NO}}^\mu), \quad I_{\text{IO}}^\tau = -(I_{\text{IO}}^e + I_{\text{IO}}^\mu). \end{aligned} \quad (5.57)$$

For the $\Delta(6 \cdot 2^2) \cong S_4$ flavor group, the value of ϱ_5 is either 0 or $\pi/2$ in the fundamental region. Notice that U_{VI} for $\varrho_5 = 0$ is equivalent to U_I . Hence the three lepton mixing angles don't subject to any constraint, and the Dirac as well as Majorana CP phases are trivial. The results of the χ^2 analysis for $\varrho_5 = \pi/2$ are collected in table 6. We display the variation of Y_B with respect to ϑ in figure 9. The observed baryon asymmetry can be obtained for certain values of ϑ except R-3rd in NO and R-1st in IO.

(VII) $G_l = \langle abc^s d^t \rangle$, $X_\nu = \rho_3(c^x d^y)$

Similar to previous cases, the lepton mixing matrix is given by, up to permutations of rows and unphysical phases,

$$\begin{aligned} U_{VII} &= \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\varrho_7} & -1 & 0 \\ e^{i\varrho_7} & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} O_{3 \times 3}(\theta_1, \theta_2, \theta_3) \widehat{X}_\nu^{-\frac{1}{2}}, \\ &= \text{diag}(-1, 1, 1) P_{132} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -e^{i\varrho_7} \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & e^{i\varrho_7} \end{pmatrix} [P_{231} O_{3 \times 3}(\theta_1, \theta_2, \theta_3)] \widehat{X}_\nu^{-\frac{1}{2}}, \end{aligned} \quad (5.58)$$

with

$$\varrho_7 = \frac{2s - t + 2x - y}{n} \pi. \quad (5.59)$$

Comparing Eq. (5.58) with Eq. (5.26), we can see this case gives rise to the same mixing pattern and the baryon asymmetry Y_B as case III if all possible row permutations are taken into account.

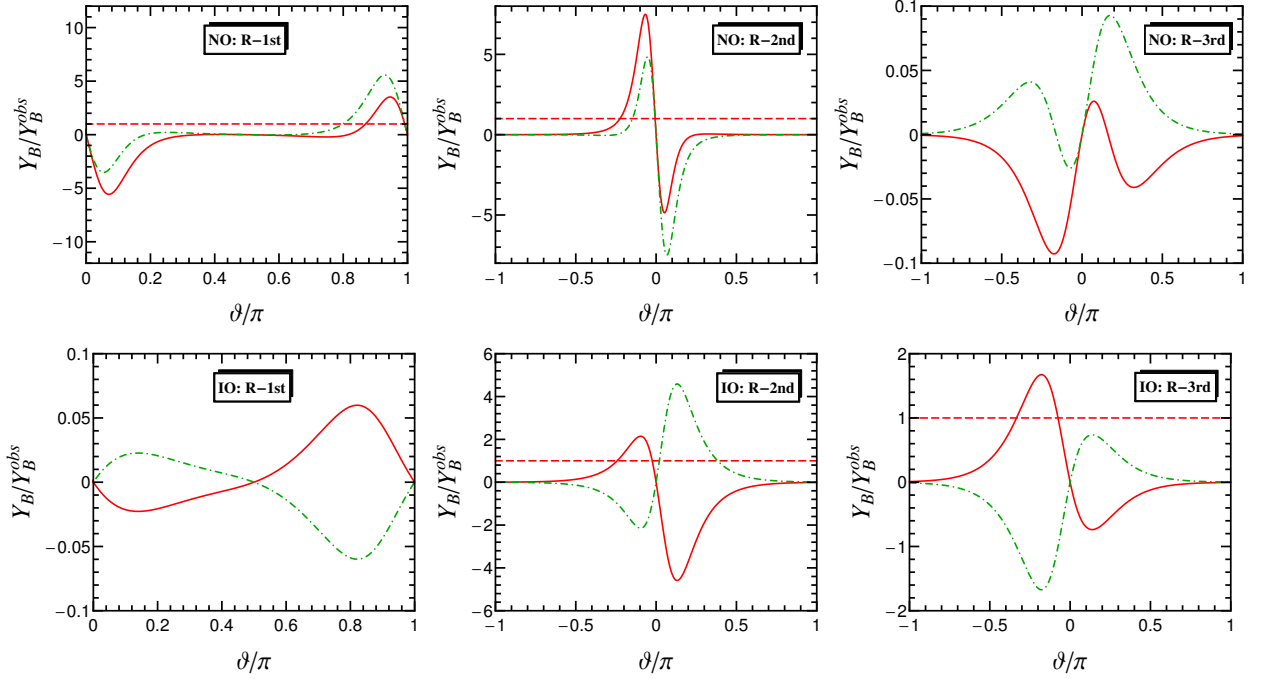


Figure 9: Y_B/Y_B^{obs} as a function of the parameter ϑ for the mixing pattern U_{VI} with $\varrho_5 = \pi/2$, where we choose the RH neutrino mass $M_1 = 5 \times 10^{11}$ GeV. The red solid and green dash-dotted lines correspond to the two best fitting points shown in table 6 one by one. The horizontal red dashed line represents the experimental measured value Y_B^{obs} . The neutrino mass spectrum is NO and IO in the first row and the second row respectively. The panels in the left, middle and right columns are for the three admissible forms of the R -matrix such as R-1st, R-2nd and R-3rd respectively.

(VIII) $G_l = \langle abc^s d^t \rangle$, $X_\nu = \rho_3(bc^x d^{-x})$

In this case, we find that the PMNS mixing matrix takes the form

$$\begin{aligned}
 U_{VIII} &= \frac{1}{2} \begin{pmatrix} -\sqrt{2} & -ie^{i\varrho_8} & e^{i\varrho_8} \\ \sqrt{2} & -ie^{i\varrho_8} & e^{i\varrho_8} \\ 0 & i\sqrt{2}e^{-i\varrho_8} & \sqrt{2}e^{-i\varrho_8} \end{pmatrix} O_{3 \times 3}(\theta_1, \theta_2, \theta_3) \hat{X}_\nu^{-\frac{1}{2}} \\
 &= \frac{1}{2} \begin{pmatrix} -\sqrt{2} & -i & 1 \\ \sqrt{2} & -i & 1 \\ 0 & i\sqrt{2} & \sqrt{2} \end{pmatrix} O_{3 \times 3}(\theta_1 - \varrho_8, \theta_2, \theta_3) \hat{X}_\nu^{-\frac{1}{2}}, \quad (5.60)
 \end{aligned}$$

with

$$\varrho_8 = \frac{2s - t + 3x}{n} \pi. \quad (5.61)$$

Obviously the value of ϱ_8 is irrelevant since it can be absorbed into the free parameter θ_1 . Furthermore, the six possible row permutations lead to three independent mixing patterns which can be chosen as

$$U_{VIII,1} = U_{VIII}, \quad U_{VIII,2} = P_{132}U_{VIII}, \quad U_{VIII,3} = P_{312}U_{VIII}. \quad (5.62)$$

The reason is because U_{VIII} fulfills the equality

$$P_{213}U_{VIII}(\varrho_8, \theta_1, \theta_2, \theta_3) = U_{VIII}(\varrho_8, \theta_1, -\theta_2, -\theta_3) \text{diag}(-1, 1, 1). \quad (5.63)$$

For the mixing matrix $U_{VIII,1}$, after the parameter θ_1 is shifted into $\theta_1 + \varrho_8$, we can read off the

mixing parameters as

$$\begin{aligned}
\sin^2 \theta_{13} &= \frac{1}{8} \left(3 - \cos 2\theta_2 - 2\sqrt{2} \cos \theta_1 \sin 2\theta_2 \right), \\
\sin^2 \theta_{12} &= \sin^2 \theta_3 + \frac{2 (\cos 2\theta_3 + \sqrt{2} \sin \theta_1 \cos \theta_2 \sin 2\theta_3)}{5 + \cos 2\theta_2 + 2\sqrt{2} \cos \theta_1 \sin 2\theta_2}, \\
\sin^2 \theta_{23} &= \frac{3 - \cos 2\theta_2 + 2\sqrt{2} \cos \theta_1 \sin 2\theta_2}{5 + \cos 2\theta_2 + 2\sqrt{2} \cos \theta_1 \sin 2\theta_2}, \\
J_{CP} &= \frac{1}{32\sqrt{2}} [4 \sin \theta_1 \sin 2\theta_2 \cos 2\theta_3 - \cos \theta_1 (\cos \theta_2 + 3 \cos 3\theta_2) \sin 2\theta_3]. \quad (5.64)
\end{aligned}$$

The CP invariants $I_{\text{NO}, \text{IO}}^\alpha$ ($\alpha = e, \mu, \tau$) turn out to take the form

$$\begin{aligned}
I_{\text{NO}}^e &= \frac{1}{4} \left[\cos \theta_2 \cos \theta_3 + \sqrt{2} (\sin \theta_1 \sin \theta_3 - \cos \theta_1 \sin \theta_2 \cos \theta_3) \right], \\
I_{\text{NO}}^\mu &= \frac{1}{4} \left[\cos \theta_2 \cos \theta_3 - \sqrt{2} (\sin \theta_1 \sin \theta_3 - \cos \theta_1 \sin \theta_2 \cos \theta_3) \right], \\
I_{\text{IO}}^e &= \frac{1}{4} \left(\sin \theta_2 + \sqrt{2} \cos \theta_1 \cos \theta_2 \right), \quad I_{\text{IO}}^\mu = \frac{1}{4} \left(\sin \theta_2 - \sqrt{2} \cos \theta_1 \cos \theta_2 \right), \\
I_{\text{NO}}^\tau &= -(I_{\text{NO}}^e + I_{\text{NO}}^\mu) = -\frac{1}{2} \cos \theta_2 \cos \theta_3, \quad I_{\text{IO}}^\tau = -(I_{\text{IO}}^e + I_{\text{IO}}^\mu) = -\frac{1}{2} \sin \theta_2. \quad (5.65)
\end{aligned}$$

Our numerical results for this case are summarized in table 6, and the variation of Y_B as a function of ϑ is showed in figure 10. From the expressions of the CP asymmetry ϵ_α and the washout mass \tilde{m}_α , we can see that the final baryon asymmetry Y_B has the following symmetry properties:

$$\begin{aligned}
Y_B(\vartheta, \theta_1 = \pi, \theta_2, \theta_3) &= -Y_B(\vartheta, \theta_1 = \pi, \theta_2, \pi - \theta_3) \\
&= Y_B(-\vartheta, \theta_1 = 0, \pi - \theta_2, \theta_3) = -Y_B(-\vartheta, \theta_1 = 0, \pi - \theta_2, \pi - \theta_3), \quad \text{for NO}, \quad (5.66)
\end{aligned}$$

$$\begin{aligned}
Y_B(\vartheta, \theta_1 = \pi, \theta_2, \theta_3) &= -Y_B(-\vartheta, \theta_1 = \pi, \theta_2, \pi - \theta_3) \\
&= Y_B(\vartheta, \theta_1 = 0, \pi - \theta_2, \theta_3) = -Y_B(-\vartheta, \theta_1 = 0, \pi - \theta_2, \pi - \theta_3), \quad \text{for IO}. \quad (5.67)
\end{aligned}$$

Thus the coincidence of two pair of curves in IO case can be easily understood from Eq. (5.67). The second mixing matrix $U_{V\text{III},2}$ is related to $U_{V\text{III},1}$ through the permutation of the second and third rows. As a consequence, the expressions for θ_{12} and θ_{13} coincide with Eq. (5.64), the overall sign of J_{CP} is reversed, while the atmospheric mixing angle θ_{23} changes into

$$\sin^2 \theta_{23} = \frac{4 \cos^2 \theta_2}{5 + \cos 2\theta_2 + 2\sqrt{2} \cos \theta_1 \sin 2\theta_2}, \quad (5.68)$$

Moreover the rephase invariants can be obtained from Eq. (5.65) by interchanging $I_{\text{NO}, \text{IO}}^\mu$ and $I_{\text{NO}, \text{IO}}^\tau$. The corresponding results of the χ^2 analysis are listed in table 6, and the predictions for the matter-antimatter asymmetry Y_B are displayed in figure 11. Finally we proceed to the third mixing matrix $U_{V\text{III},3}$, we can extract the following results for the mixing angles

$$\sin^2 \theta_{13} = \frac{1}{2} \cos^2 \theta_2, \quad \sin^2 \theta_{12} = \frac{1}{2} + \frac{\cos^2 \theta_2 \cos 2\theta_3}{3 - \cos 2\theta_2}, \quad \sin^2 \theta_{23} = \frac{1}{2} - \frac{\sqrt{2} \cos \theta_1 \sin 2\theta_2}{3 - \cos 2\theta_2}, \quad (5.69)$$

which implies

$$\left| \sin^2 \theta_{12} - \frac{1}{2} \right| \leq \frac{1}{2} \tan^2 \theta_{13}, \quad \left| \sin^2 \theta_{23} - \frac{1}{2} \right| \leq \tan \theta_{13} \sqrt{1 - \tan^2 \theta_{13}}. \quad (5.70)$$

Hence the experimental data [4] on θ_{13} and θ_{12} can not be accommodated simultaneously without higher order corrections for this mixing matrix.

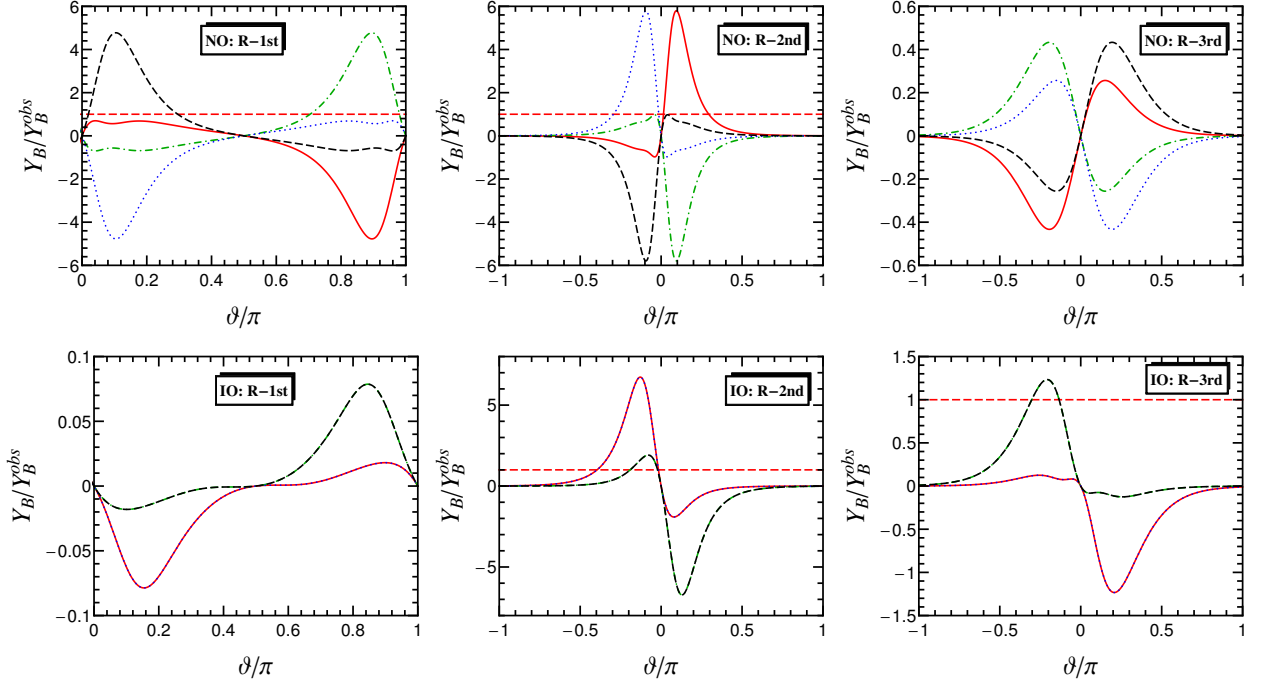


Figure 10: Y_B/Y_B^{obs} as a function of the parameter ϑ for the mixing pattern $U_{VIII,1}$, where we choose the RH neutrino mass $M_1 = 5 \times 10^{11}$ GeV. The red solid, green dash-dotted, blue dotted and black dashed lines correspond to the four best fitting points shown in table 6 one by one. The horizontal red dashed line represents the experimental measured value Y_B^{obs} . The neutrino mass spectrum is NO and IO in the first row and the second row respectively. The panels in the left, middle and right columns are for the three admissible forms of the R -matrix such as R-1st, R-2nd and R-3rd respectively.

6 Conclusions

The smallness of neutrino masses can be naturally explained by the seesaw mechanism in which two or three RH neutrinos are added in the SM. The 2RHN model can be regarded as the limiting case of the three RH neutrino model in which one of the RH neutrinos is very heavy. The 2RHN model is more predictive than the three RH neutrino model because the number of parameters is greatly reduced. One remarkable feature is that the lightest neutrino is massless in the 2RHN model. Leptogenesis is a natural cosmological consequence of the seesaw mechanism, and it provides a simple explanation for the matter-antimatter asymmetry of the Universe.

Finite discrete flavor symmetry and CP symmetry which are broken to distinct subgroups in the charged lepton and neutrino sectors, is a quite powerful approach to explain the lepton mixing angles and CP violation phases. Other phenomenons involving CP phases, such as neutrinoless double beta decay and leptogenesis, are also subject to strong constraint in this approach. In the present work, we study the interplay between residual symmetry and leptogenesis in the 2RHN model, and we assume that the scale of flavor symmetry breaking is above the leptogenesis scale. In our method, only the residual symmetry is assumed, and we do not need to consider the possible dynamics which realizes the residual symmetry.

Without loss of generality we work in the basis in which both the charged lepton and RH neutrino mass matrices are diagonal. If two residual CP transformations or a cyclic residual flavor symmetry arising from the original flavor and CP symmetries are preserved by the seesaw Lagrangian, we find that each row of the R -matrix would have only one nonzero entry which is equal to ± 1 . Hence the baryon asymmetry would be predicted to be zero without invoking subleading order corrections. In most discrete flavor symmetry models [13–15], the leading order remnant

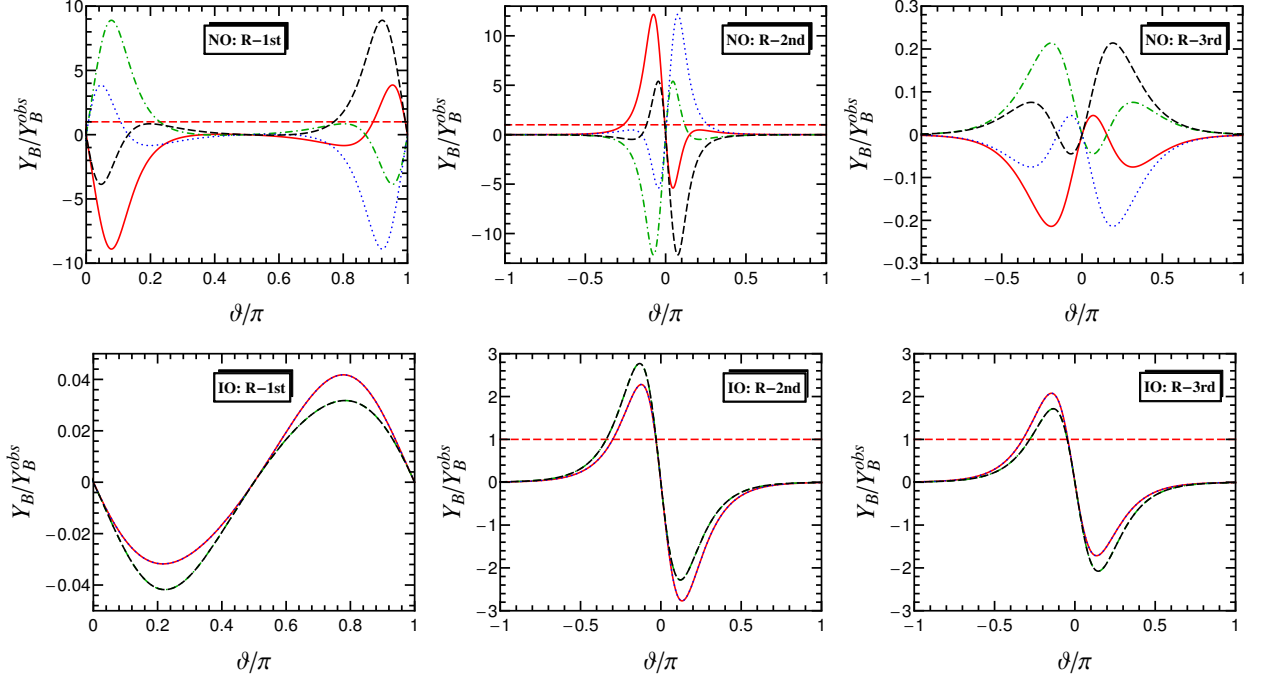


Figure 11: Y_B/Y_B^{obs} as a function of the parameter ϑ for the mixing pattern $U_{VIII,2}$, where we choose the RH neutrino mass $M_1 = 5 \times 10^{11}$ GeV. The red solid, green dash-dotted, blue dotted and black dashed lines correspond to the four best fitting points shown in table 6 one by one. The horizontal red dashed line represents the experimental measured value Y_B^{obs} . The neutrino mass spectrum is NO and IO in the first row and the second row respectively. The panels in the left, middle and right columns are for the three admissible forms of the R -matrix such as R-1st, R-2nd and R-3rd respectively.

symmetries of the neutrino and charged lepton sectors are generally broken by the higher dimensional flavon cross operators. Hence the tiny baryon asymmetry Y_B is expected to be naturally reproduced.

If a single residual CP transformation is preserved in the neutrino sector, the lepton mixing matrix contains three real free parameter $\theta_{1,2,3}$ in the range of $[0, \pi)$, the R -matrix is found to depends on only one real parameter ϑ and it can take three viable forms summarized in Eq. (3.8). Each entry of the R -matrix is real or pure imaginary in this case, consequently the total CP asymmetry ϵ_1 vanishes unless the non-leading contributions are taken into account in a concrete model. Hence in this paper we discuss the flavored thermal leptogenesis in which the interactions mediated by the τ lepton Yukawa couplings are in equilibrium, and the lightest RH neutrino mass is typically in the interval of $10^9 \text{ GeV} \leq M_1 \leq 10^{12} \text{ GeV}$. Then the baryon asymmetry is generated uniquely by the CP phases in the PMNS mixing matrix in this scenario. Therefore the observation of low energy leptonic CP violating phases would imply the existence of a baryon asymmetry. Moreover, we have performed a general analysis of leptogenesis in the 2RHN model with a residual CP transformation. For illustration, the numerical results of Y_B for $\delta = 0, -\pi/2$ are presented, as shown in figures 1, 2 and 3.

We have performed a comprehensive study in which the single remnant CP transformation originates from the CP symmetry compatible with the $\Delta(6n^2)$ flavor group which is broken to an abelian subgroup in the charged lepton sector. All possible residual symmetries and the resulting predictions for lepton flavor mixing and leptogenesis are studied. We find there are in total eight possible cases (from case I to case VIII). The case I and case IV give rise to the same lepton mixing pattern and the same results for leptogenesis. The cases III and VII are also the same after the shift of the free parameters $\theta_{1,2,3}$ is taken into account. The PMNS matrix in cases I and IV is real up to

the CP parity of the neutrino states. As a consequence, although the experimental data on mixing angles can be accommodated in these cases, all the leptogenesis CP asymmetries are vanishing and a net baryon asymmetry can not be generated without corrections. For the remaining cases, the observed vmatter/antimatter asymmetry could be reproduced except for R-3rd with NO spectrum and R-1st of IO. Moreover, we find that small $\Delta(6n^2)$ group (e.g., $n = 2, 3, 4$ etc) can describe the experimentally measured values of the mixing angles for certain choices of the parameter values. Our approach is very general and model independent, and the results of this paper should be helpful to discuss the phenomenology of leptogenesis in a specific 2RHN model based on flavor and CP symmetries.

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